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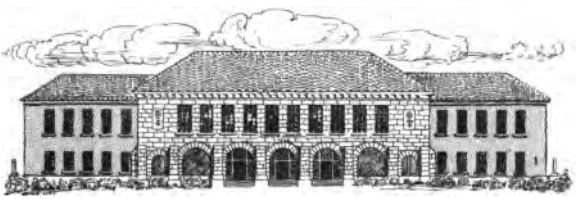

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MANUAL

OF

MENSURATION,

FOR

COMMON AND HIGH SCHOOLS.

BY

HENRY H. HUTTON, M. A.,

Principal of the High School,

WAVERLY, N. Y.

SYRACUSE, N. Y.:
C. W. BARDEEN, PUBLISHER,
1882.

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PREFACE.

The formulas of this little work have been collected from all sources. Special acknowledgements, however, are due to the very excellent work on Mensuration connected with the Irish National Series of School Books, for the demonstration of many of the formulas.

Originality is here claimed mainly for the mode of presentation. Having used the formulas here given, for many years, with most gratifying results, the author has thought it worth while to offer them in this form.

A bright class can finish the book in one term. It is suggested that the formulas be thoroughly memorized, especially the fundamental ones. It is not intended that the pupil shall be made to understand the reason why, except so far as the teacher may see his opportunity to make it plain.

It is *intended for a mechanical drill*, to present the Art, not the Science. The demonstrations referred to in the Conspectus of Formulas, and given in the Supplement, are for the teacher's use. Attention is called to an easy system of Mnemonics, to be found on page 115.

If this little work shall add something to the interest of the school-room, and aid in taking away from the same the reproach of want of practicality so often brought when pupils fail to give practical answers at home, the author will feel well repaid for the time and toil he has given.

H. H. H.

Waverly, N. Y., Aug. 27, 1881.

MANUAL OF MENSURATION.

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A MANUAL

OF

MENSURATION.

1. **Geometry** demonstrates the properties of all kinds of magnitude or extension; as solids, surfaces, lines, and angles.

2. **Mensuration,** applying the deductions of geometry, consists in determining the areas of surfaces and the volumes of solids, from their linear dimensions.

It is of two kinds, viz: mensuration of surfaces, and mensuration of solids.

MENSURATION OF SURFACES.

3. The **area** of any plane figure is the space contained within its boundaries, and is estimated by the number of square miles, square perches, square yards, etc., which it contains.

4. A **triangle** is a figure contained by three straight lines: when these three are equal it is said to be equilateral; when two are equal,

isosceles; when the three sides are unequal, the triangle is scalene.

TRIANGLES.

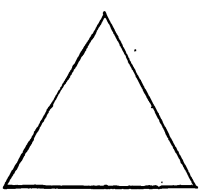


Fig. 1. Equilateral.

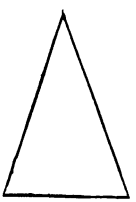


Fig. 2. Isosceles.

Fig. 3. Scalene.

5. A triangle is said to be **right, obtuse** or **acute,** according as it contains a right-angle, an obtuse angle, or all acute angle.

A **right-angle** is formed by the meeting of two straight lines perpendicular to each other. An **obtuse angle** is greater than a right-angle. An **acute angle** is less than a right-angle.

Fig. 4. Right-angled.

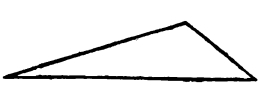


Fig. 5. Obtuse-angled.

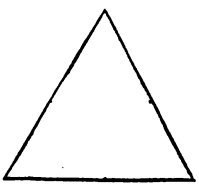


Fig. 6. Acute-angled.

6. In figure four the side $\overline{B\ C}$, opposite the right-angle is called the **hypothenuse**; $\overline{A\ C}$ is the **base**, and $\overline{A\ B}$, the **perpendicular**, or **hight.**

7. If a straight line be let fall from the right-angle of a right-angled triangle, perpendicular to the hypothenuse, the parts into which it divides it are called **segments.** Thus :—

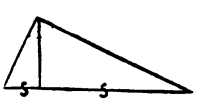


Fig. 7.

TRIANGULAR FIGURES.

8. In the mensuration of triangular figures let A=area, b=base, p=perpendicular, or hight, h=hypothenuse, p′=perpendicular from right-angle on the hypothenuse, s and s′, the greater and lesser segment, respectively, into which the perpendicular from the right-angle of a right-

angled triangle divides the hypothenuse. Then we shall have the following *formulas*:—

SYNOPSIS OF FORMULAS.

9. $A = \frac{bp}{2}$ (I); $b = \frac{2A}{p}$ (II); $p = \frac{2A}{b}$ (III); $A = \sqrt{m\,(m-a)\,(m-b)\,(m-c)}$, (IV). NOTE. In this formula a, b and c represent the sides of a triangle respectively, and m, half their sum.

$A = \frac{b^2}{4}\sqrt{3} = .433\ b^2$ (V); $h = \sqrt{b^2+p^2}$ (VI);

$b = \sqrt{h^2-p^2}$ (VII); $p = \sqrt{h^2-b^2}$ (VIII);

$s' = \frac{p^2}{h}$ (IX); $s = \frac{b^2}{h}$ (X); $p' = \sqrt{ss'}$ (XI).

PROBLEM I.

Given the base and perpendicular of a triangle, to find its area.

Rule I. Take half the product of the base and perpendicular: or, $A = \frac{bp}{2}$.

EXERCISE I.

1. Find the area of a triangle whose base is 16 and perpendicular 11.

2. Find the area of a triangle whose base is 9 inches and perpendicular 5 inches.

3. Find the area of a triangle whose base is 24 chains and hight 91 chains.

4. How many square yards in a triangle whose base is 13 yards and perpendicular 10 yards?

5. The base of a triangle is 15 ft. 6 in., and

the perpendicular is 12 ft. 7 in.: what is its area?

6. What is the hight of a triangle whose area is 64 sq. inches, and base 12 inches?

7. What is the base of a triangle whose area is 84 sq. ft., and perpendicular 8 feet?

8. A triangular field measures 40 perches on one side; and the shortest distance from the side to the angle opposite it is 18 perches; required its area.

PROBLEM II.

Given the sides of any triangle to find its area.

Rule II. From half the sum of the three sides subtract the three sides separately; then multiply the half sum and the three remainders together, and extract the square root of the product; or,—

$$A = \sqrt{m\,(m-a)\,(m-b)\,(m-c)}.$$

EXERCISE. II.

1. Find the area of a triangle whose sides are respectively 3, 4, and 5.

2. How many square yards are in a triangle whose three sides are 13, 14 and 15 feet?

3. How many acres of land in a triangular

plot of ground, whose three sides are respect ively 7perches 8 perches, and 11 perches?

4. Find the area of a triangle whose three sides are 10, 12, and 14 links respectively.

5. Find the area of a triangle whose sides are 4, 6, and 8 yards.

6. The area of a triangle is 600, one side is 50, the sum of sides is 120, the other two sides are to each other as 3 to 4, find these.

PROBLEM III.

Given the base of an equilateral triangle, to find the area.

Rule III. Multipiy a fourth of the square of the base by the square root of three; or, $A=\frac{b^2}{4}\sqrt{3}=.433\ b^2$.

EXERCISE III.

1. Find the area of an equilateral triangle whose base is 4.

2. Find the area of an equilateral triangle whose base is 1000 yards.

3. How many acres in a field of a triangular form, each of whose sides measures 70 perches?

4. The perimeter of an equilateral triangle is 27 feet; what is its area?

5. Each side of a triangle measures 4 perches: what is its area?

6. The area of an equilateral triangle is 36, what is its base?

PROBLEM IV.

Given any two sides of a right-angled triangle, to find the third side.

Rule IV. The hypothenuse equals the square root of the sum of the squares of the base and perpendicular; or, $h = \sqrt{b^2 + p^2}$.

Rule V. The base equals the square root of the difference of the squares of the hypothenuse and perpendicular; or, $b = \sqrt{h^2 - p^2}$.

Rule VI. The perpendicular equals the square root of the difference of the squares of the hypothenuse and base; or $p = \sqrt{h^2 - b^2}$.

EXERCISE IV.

1. In a right-angled triangle the base is 4 and the perpendicular 3; required the hypothenuse.

2. Given the hypothenuse 5 and the base 3; required the perpendicular.

3. Given the hypothenuse 5 and the perpendicular 4; required the base.

4. The wall of a building, on the brink of a river is 120 feet high, and the breadth of the

river is 70 yards, what is the length of the cord, in feet, that will reach from the top of the building across the river?

5. A ladder 60 feet long will reach to a window 40 feet from the sidewalk on one side of the street, and by turning the ladder over to the other side of the street it will reach the plate of of a building 50 feet high; how wide is the street?

PROBLEM V.

Given the three sides of a right-angled triangle, to find the perpendicular let fall from the right-angle on the hypothenuse, and also the segments into which the hypothenuse is divided by this perpendicular.

Rule VII. The segment not adjacent to the base, or greater side of the two containing the right-angle equals the square of the perpendicular divided by the hypothenuse; or, $s' = \frac{p^2}{h}$.

Rule VIII. The segment adjacent to the base or greater side of the two containing the right-angle, equals the square of the base divided by the hypothenuse; or, $s = \frac{b^2}{h}$.

Rule IX. The perpendicular let fall from the

right-angle is equal to the square root of the product of the segments; or $p' = \sqrt{ss'}$.

EXERCISE V.

1. Find the perpendicular on the hypothenuse, and also, find the segments of the hypothenuse of a right-angled triangle whose base and perpendicular are 50 and 60.

2. Find the perpendicular on the hypothenuse of a right-angled triangle whose base and hight are respectively 35, and 30.

3. The roof of a house whose side walls are each 30 feet high, forms a right-angle at the top; now if one of the rafters is 10 feet long, and its opposite 12, required the breadth of the building, the length of the prop set upright to hold the ridge of the roof and the part of the floor at which it must be placed.

GENERAL EXERCISE IN THE MENSURATION OF TRIANGLES.

1. Find the altitude of a triangle whose area is 7196 square feet, and base 120 feet.

2. The area of an equilateral triangle is 9134 square yards, what is its base?

3. What is the hight of a triangle whose base is 750 chains, and area 50 acres?

4. Find the area of an equilateral triangle whose base is 20.

5. Find the area of a triangle whose sides are 90, 70, and 60 chains in length respectively.

6. Find the area of a triangle whose base is $12\frac{1}{2}$ chains and hight $8\frac{2}{5}$ chains.

7. What is the length of the base of a triangle whose area is 2560 square feet, and hight 40 feet?

8. What is the hight of a triangle which contains 117.5625 square yards; its base being 49 feet 6 inches in length?

9. Each of the sides of a triangular flower-bed is 25 yards long; what is its area?

10. A surveyor having lost his field-book and requiring the base of a triangular field whose contents he knew from recollection was 14 acres, and perpendicular hight 7 yards, was enabled readily to give it. What was it?

11. What is the hight of a triangle whose area is 12 and base 6?

12. The area of two similar triangular fields are 60 and 90 acres and a side of the former is 66 perches. Required the corresponding side of the latter.

13. The hight of a tree growing in the centre

of a circular island 100 feet in diameter is 160 feet, and a line extending from the top of it to the farther shore is 400 feet. What is the width of the stream, the land on each side of the stream being level?

14. A tree 80 feet in hight stands on a horizontal plane; at what hight from the ground must it be broken off, so that the top of it may fall on a point 40 feet from the bottom of the tree, the end where it is broken off resting on the stump?

NOTE. The triangle formed by the two parts of the broken tree and the distance from the bottom of the tree to where the top rests, namely, 40 feet, is similar to a right-angled triangle whose base, hight, and hypotenuse are, respectively 4, 3 and 5. Now, let it be remembered tha *similar* triangles have their like sides proportional.

15. In the ruins of Persepolis there are two columns left standing upright: the one is 64 feet above the plain, and the other 50. In a straight line between these stands a small statue, the head of which is 97 feet from the top of the higher column, and 86 feet from the top of the lower, the base of which is 76 feet from the base of the statue. Required the distance between the tops of the columns.

QUADRILATERAL FIGURES.

10. A **quadrilateral** is a polygon having four sides and four angles.

11. A **parallelogram** is a quadrilateral which has its opposite sides parallel.

12. A **rectangle,** or **oblong** is a right-angled parallelogram whose length exceeds its breadth; as Fig. 8.

Fig. 8.

13. A **square** is an equilateral rectangle; as Fig. 9.

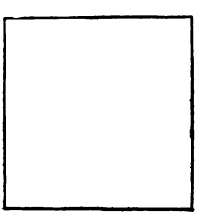


Fig. 9.

14. A **trapezoid** is a four-sided figure having two of its opposire sides parallel; as Fig. 10.

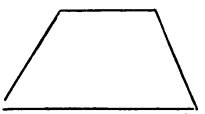


Fig. 10.

15. A **trapezium** is a figure included by four straight lines no two of which are parallel to each other; as Fig. 11.

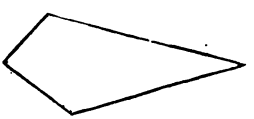


Fig. 11.

In the mensuration of quadrilateral figures let A = area, b = base, p = perpendicular and d = diagonal; also, let b and b′ = parallel sides of trapezoid and 1 and 1′ = perpendiculars from the diagonal of a trapezium to the opposite angles. Then we have the following formulas:

SYNOPSIS OF FORMULAS.

For square. $A = b^2$ (XII); $b = \sqrt{A.}$ (XIII).

$A = \frac{d^2}{2}$ (XIX); $d = \sqrt{2A.}$ (XV).

For Rectangle or Parallelogram. $A = bp$ (XVI); $b = \frac{A}{p}$ (XVII); $p = \frac{A}{b}$ (XVIII).

For Rectangle only. $A = \sqrt{(d+b)(d-b)}$ (XIX).

For Trapezoid. $A = \frac{p}{2}(b+b')$ (XX).

For Trapezium. $A = \frac{d}{2}(l+l')$ (XXI).

PROBLEM VI.

The side of a square being given to find its area.

Rule X. Square the side for the area; or, $A = b^2$.

EXERCISE VI.

1. Let the side of a square be six; what is its area?

2. What is the area of a square whose side is 15 chains?

3. What is the area of a square whose side is 7 feet 9 inches?

4. What is the area of a square whose side is 916 yards?

PROBLEM VII.

The diagonal of a square being given to find its area.

Rule XI. Take half the square of the diagonal for the area; or, $A = \frac{d^2}{2}$.

EXERCISE VII.

1. The diagonal of a square is 5; what is its area?

2. Find the area of a square whose diagonal is 12.

PROBLEM VIII.

Given the base and perpendicular of a rectangle or parallelogram, to find its area.

Rule XII. Multiply the base by the perpendicular; or, $A = bp$.

EXERCISE VIII.

Find the area of a rectangle whose base is 11, and side 18.

2. What is the superficial contents of a plank whose length is five feet six inches, and breadth, seven feet eight inches.

3. A rectangular field is 170 rods in length, and 15 rods in width; what is its area?

4. What is the area of a parallelogram whose base is three yards and hight 2 feet 3 inches?

4. What is the area of a parallelogram whose base is 7 feet 6 inches and hight 3 feet 4 inches?

5. A field in the form of a parallelogram is 76 chains in length, and a path proceeding at right-angles from this side to the opposite side measures 5.7 chains; what is its area?

PROBLEM IX.

Given the opposite parallel sides of a trape-

zoid and the perpendicular distance between them to find the area.

Rule XIII. Multiply the sum of the parallel sides by half the hight; or, $A = \frac{p}{2}(b + b')$.

EXERCISE IX.

1. In a trapezoid the two parallel sides are 20 and 18 inches, respectively, and the distance between them is 14 inches; what is the area?

2. What is the area of a trapezoid whose parallel sides are 100 and 2000 links, and the distance between them 620 links?

PROBLEM X.

Given the diagonal of a trapezium and the perpendiculars from the diagonal to the opposite angles to find the area.

Rule XIV. Multiply the sum of the perpendiculars by half the diagonal: or, $A = \frac{d}{2}(l + l')$.

EXAMPLE. In the trapezium a, b, c, d, let the diagonal $\overline{a\,c} = 22$, and let the perpendicular $b\,f = 16$, and $\overline{e\,d} = 8$, then:—

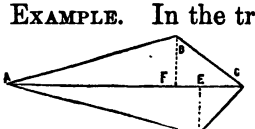


Fig. 12.

$A = \frac{d}{2}(b + b') = \frac{22}{c}(16 + 8) = 11 \times 24 = 264$; Ans.

EXERCISE X.

1. What is the area of a trapezium whose diagonal is 10 perches, and the perpendiculars from it to the opposite angles are 40 and 12 perches, respectively?

GENERAL EXERCISES IN THE MENSURATION OF QUADRILATERALS.

1. What is the base of a square whose area is 56 a., 17 r., 25 perches?

2. What is the diagonal of a square whose area is 312.5 furlongs?

3. What is the diagonal of a rectangle whose area is 20 and base 5?

4. Find the area of a rectangular field whose base is 25 perches and diagonal 40 perches.

5. Find the distance between the sides of a parallelogram whose base is 900 yards and area 6 a., 2 r., 28 per., 17 yds.

6. The base of a parallelogram is 12 feet and its height 8; what is its area?

7. The largest of the Egyptian pyramids is square at the base and measures 693 feet on a side; how much ground does it cover?

8. What is the area of a garden in the form of a trapezium, of which the four sides are 328, 356, 572 and 298 feet, and the diagonal

drawn from the vertex of the angle contained by the first two sides to its opposite angle is 598 feet?

POLYGONS.

18. Polygons are of two kinds, **regular** and **irregular.**

19. A regular polygon is one whose sides are all equal to one another, as, also, are its angles; and they are named from the number of their sides:—thus, a **five-sided** polygon is called a **pentagon;** a six-sided one, a **hexagon;** a seven-sided one, a **heptagon;** an eight-sided one, an **octagon;** a nine-sided one, a **nonagon;** a ten-sided one, a **decagon;** an eleven-sided one, an **undecagon;** a twelve-sided one, a **dodecagon.**

20. An irregular polygon is one whose sides are not all equal.

21. **The Apothem** of a regular polygon is a perpendicular from its centre on any of its sides; as $\overline{A\,C}$, figure XIII.

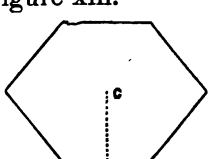


Fig. 13.

TABLE I.

Showing **apothem** and **area** of regular polygons when side equals unity.

NAMES OF POLYGON.	SIDES	APOTHEM.	AREA.
Triangles	3	0.2886751	0.4330127
Square	4	0.5	1.
Pentagon	5	0.688191	1.7204774
Hexagon	6	0.8660254	2.5980762
Heptagon	7	1.0382607	3.6339124
Octagon	8	1.2071068	4.8284271
Nonagon	9	1.3737387	6.1818242
Decagon	10	1.5388418	7.6942088
Undecagon	11	1.7028436	9.3656399
Dodegagon	12	1.8660254	11.1961524

22. To find the **apothem** of any regular polygon we have only to multiply the corresponding **apothem** in the table by the side of the given polygon. The **area**, also, may be found by multiplying the corresponding area in the table by the square of the given side.

23. But, let a = apothem when side is unity, s = side, and n = number of sides; then we have the following formulas:

24. $A = \frac{a\,n\,s}{2}$ (XXII); $s = \frac{2\,A}{a\,n}$ (XXIII); $n = \frac{2\,A}{a\,s}$ (XXIV).

PROBLEM XI.

Given the side of a regular polygon to find its area.

Rule XV. Take half the continued product of its apothem, side, and number of sides; or, $A = \frac{1}{2}\,a\,n\,s$.

EXAMPLE. Find the area of a regular pentagon whose side is 10 feet.

SOLUTION.

From table I, the apothem of a pentagon whose side is unity is 0.688191, and, section 22, this multiplied by $10 = .688191 \times 10 = 6.88191 =$ apothem of pentagon whose side is 10. Then, by formula XXII, we have—$A = \frac{1}{2}\,a\,n\,s = \frac{1}{2} \times .688191 \times 5 \times 10 = 17.20475$ sq. ft., Ans.

EXERCISE XI.

1. What is the area of a regular octagon whose side is 10 inches.

2. What is area of a regular dodecagon whose side is 40 feet?

3. What is the length of a side of a regular hexagon whose area is 4156.915 square yards?

4. Find the number of sides in a regular polygon whose area is 123.1072 square yards; its apothem being 6.15536, and side 4 yards?

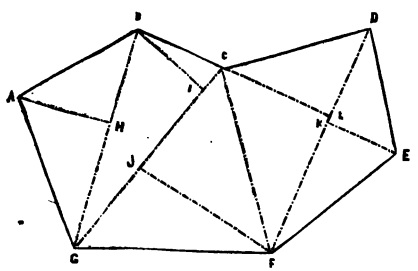
5. What is the area of a park in the form of an octagon whose side is 12 chains and apothem 14.485 chains?

PROBLEM XII.

Give **data** sufficient to solve triangles and trapeziums, to find the area of an irregular polygon.

Rule XVI. Divide the figure into triangles and trapezium, and find the area of each separately; the sum of these areas will be the area of the polygon.

EXAMPLE. Find the area of the irregular polygon a, b, c, d, e, f, g, in which,—



g b = 29
g c = 30
f d = 35
k e = 14
l c = 20
j f = 26
b i = 12
a h = 8

Fig. 14.

Let the diagonals and perpendidulars necessary to the division of the plot and the measurement of the several triangles and trapezium, be represented by the dotted lines; then,

Triangle g a b = ½ g b × a h = ½ × 8 × 29 = 116 (1);
" g b c = ½ g c × b i = ½ × 30 × 12 = 180 (2);
" g c f = ½ g c × j f = ½ × 30 × 26 = 390 (3);
Trapezium f c d e = ½ f d (c l + k e) = ½ × 35 (20 + 14) = 595 (4);

And the sum of the four results equals 116 + 180 + 390 + 595 = 1281, Ans.

EXERCISE XII.

1. In the accompanying figure, there are given in perches,—

A X = 15	A P = 20	F R = 10
X R = 18	P S = 22	E T = 14
R T = 18	S D = 17	B P = 20
T D = 12	G X = 5	C S = 12

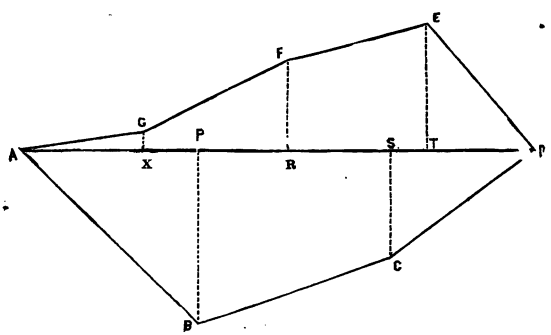


Fig. 15.

What is the area?

CIRCLES.

25. A circle is a plane figure bounded by one uniformly curved line, all of the points in

which are at the same distance from a certain point within called the centre; as M. Fig 16.

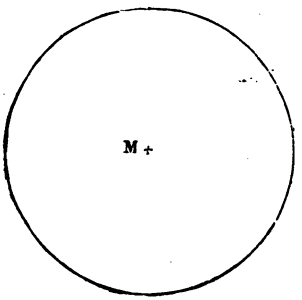


Fig. 16.

26. The circumference of a circle is the curved line about it as A B C, fig. 17.

27. The diameter of a circle is a line passing through the centre and terminated at both extremities in the circumference; as A C B, fig. 18, (*p.* 32.)

28. The radius of a circle is half of a diameter, or, is a straight line joining the centre of a circle with the circumference, as C B, fig. 18, (*p.* 32.)

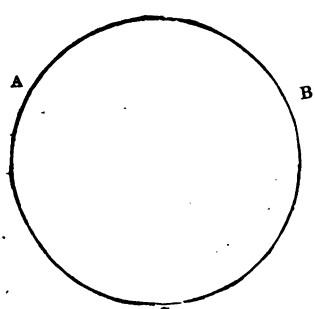


Fig. 17.

29. An arc is any part of the circumference, as A G B, fig. 19, (*p.* 33.)

30. A chord of a circle is any straight line joining the extremities of any arc; as A B, fig. 19, (*p.* 33.)

31. A semicircle is a figure contained by the diameter and the part of the circumference cut off by the diameter; as A G B, fig. 18.

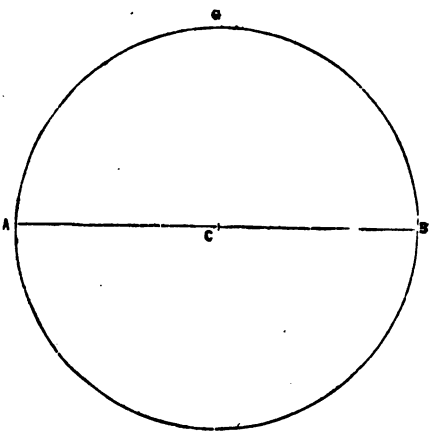


Fig. 18.

32. A segment of a circle is the figure contained by a chord and the arc of a circumference cut off by the chord; as N, or O, fig 19.

33. A sector of a circle is the figure contained by an arc of the circumference, and the two radii joining the extremities with the centre of the circle; as A O C, fig. 20.

34. A lune is the figure contained between the circular-arcs of two dissimilar circular seg-

ments which have a common chord; as fig. 21.

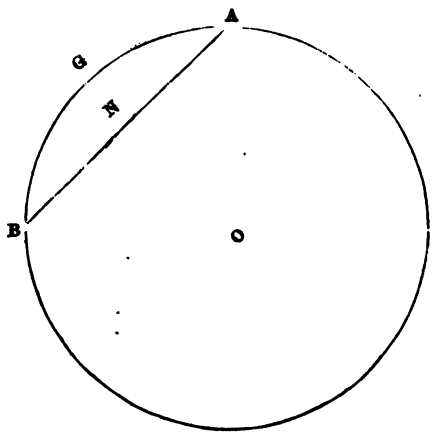


Fig. 19.

35. A Degree is the 360th part of the circumference of a circle.

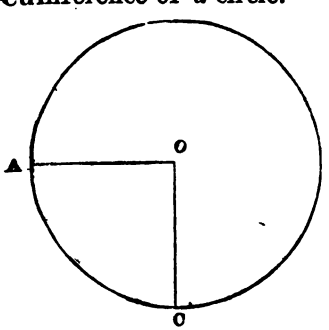


Fig. 20.

36. Concentric circles are such as have a common centre; as A and B, and an annulus is the space or ring between the two circles; as a b c, fig. 22.

37. Let d = diameter, r = radius, c = circumference, and $\pi = 3.1416$: then, for the mensuration of circles we have the following formulas:

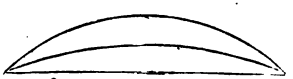


Fig. 21.

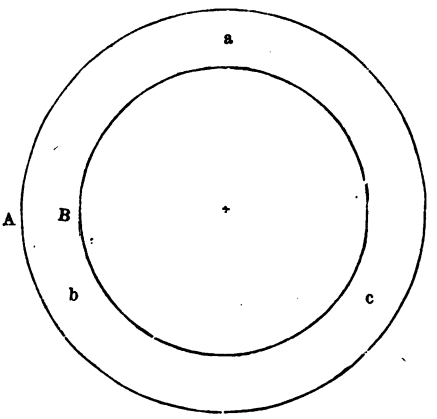


Fig. 22.

SYNOPSIS OF FORMULAS.

38. $c = \pi d$ (XXV); $d = \frac{c}{\pi} = c \times \frac{1}{3.1416} = .3183c$, (XXVI). $A = \frac{1}{4} c d$ (XXVII); whence also, $d = \frac{4A}{c}$ (XXVIII), and $c = \frac{4A}{d}$ (XXIX). $A = \pi r^2$ (XXX); $r = \sqrt{\frac{A}{\pi}} = \sqrt{A \times .3183}$ (XXXI). $A = d^2 \times .7854$,

because $d^2 = 4\ r^2$, and $3.1416 \div 4 = .7854$ (XXXII). $A = .0796\ c^2$, since $d = .3183\ c$ (XXXIII); hence $d^2 = (.3183)^2 \times c^2$ (XXXIV).

NOTE. The symbol π is the Greek letter called pi.

PROBLEM XIII.

Given the diameter of a circle to find the circumference.

RULE. Multiply the diameter by 3.1416; or, $c = \pi\ d$.

PROBLEM XIV.

Given the circumference of a circle to find its diameter.

RULE. Divide the circumference by 3.1416; or, $d = \frac{c}{\pi}$.

PROBLEM XV.

Given the circumference and diameter of a circle, to find the area.

Rule I. Take a fourth of the product of the circumference by the diameter; or, $A = \frac{c\ d}{4}$.

Rule II. Multiply the square of the radius by 3,1416; or, $A = \pi\ r^2$.

Rule III. Multiply the square of the diameter by .7854; or, $A = d^2 \times .7854$.

Rule IV. Multiply the square of the circumference by .0796; or, $A = .0796\ c^2$.

EXERCISE XIII.

Involving Problems, XIII, XIV, XV.

1. The diameter of a circle is 15, what is the circumference?

2. The circumference of a circle is 80, what is its diameter?

3. The diameter of the earth is 7912 miles, what is its circumference?

4. What is the area of a circle whose diameter is 12?

5. How many square yards are there in a circle whose diameter is $1\frac{1}{3}$ yards?

6. The circumference of a fish pond is 56 chains, what is the area?

7. What is the area of a quadrant, the radius being 100 inches?

8. Required the length in yards of a cord fastened to a stake at one end and to a cow's horns at the other, so as to let her feed on an acre of grass and no more?

9. What is the area of a semicircle whose radius is 20?

10. What is the diameter of a circle whose circumference is 78.54 yards?

11. The bottom of a circular cistern is 5 feet in diameter, what is its area?

12. Find the area of a circle whose radius is 35.

CIRCULAR ANNULUS.

39. Let d = diameter of the greater and d′ = that of the lesser circle, and let c and c′ = their circumferences, respectively; then we have the following

SYNOPSIS OF FORMULAS.

40. $A = \frac{\pi}{4}(d+d')\ (d-d')$ (XXXV). $A = .0796\ (c+c')\ (c-c')$ (XXXVI). $A = \frac{1}{4}(c+c')\ (d-d')$ (XXXVII).

PROBLEM XVI.

Given the diameter or circumference of two concentric circles, to find the annulus.

Rule I. Multiply the product of the sum and difference of the two diameters by the fourth part of 3.1416; or, $A = \frac{\pi}{4}(d+d')\ (d-d')$.

Rule II. Multiply the product of the sum and difference of the circumferences by the decimal .0796; or $A = .0796\ (c+c')\ (c-c')$.

Rule III. Take a fourth part of the product arising from multiplying the sum of the circumferences by the differences of the diameters; or, $A = \frac{1}{4}(c+c)\ (d-d')$.

EXERCISE XIV.

1. What is the area of a circular annulus when the diameters are 40 and 30?

2. What is the area of an annulus contained between two circles whose circumferences are 20 and 50 feet?

3. What is the area of an annulus, the diameters of the bounding circles being 20 and 50, and the circumferences 62.832, and 157.08?

PROBLEM XVII.

Given the length of the bounding arcs and their distance apart, to find the area of a sub-annulus.

RULE. Multiply half the sum of the bound-arcs by their distance apart.

CIRCULAR ARCS, CHORDS, AND CHORDS OF HALF-ARCS.

41. Let n = number of degrees in the arc, d = diameter of circle, l = length of arc, k = chord of whole arc, k′ = chord of half the arc, a = apothem, and h = hight of segment; then we have the following

SYNOPSIS OF FORMULAS.

42. $l = \frac{\pi n d}{360} = .008726\ n\ d$ (XXXVIII);

$k = 2\sqrt{r^2 - a^2}$ (XXXIX): $a = \frac{1}{2}\sqrt{4r^2 - k^2}$ (XL):

$k' = \sqrt{2r(r-a)}$ (XLI): $r = \frac{h'^2}{2h}$ (XLII).

PROBLEM XVIII.

Given the number of degrees of an arc and the diameter of the circle, to find the length of the arc.

RULE. Divide the continued product of 3.1416, the number of degrees in the arc, and the diameter of the circle by 360; or, $l = \frac{\pi n d}{360} = .008726\ n\ d$.

EXAMPLE. Find the length of a circular arc of 28°, the diameter of the circle being 12 inches.

SOLUTION. $l = .008726\ n\ d = .008726 \times 28 \times 12 = 2.931936$ inches, Ans.

PROBLEM XIX.

Given the apothem of an arc and the radius of the circle, to find the chord.

RULE. Take twice the square root of the difference of the squares of the radius and apothem; or, $k = 2\sqrt{r^2 - a^2}$.

EXAMPLE. Find the chord of an arc whose apothem is 12, and radius, 15.6205.

SOLUTION. $k = 2\sqrt{r^2-a^2} = 2\sqrt{15.6205^2 - 12^2} = 2\sqrt{244-144} = 2\sqrt{100} = 2 \times 10 = 20$. Ans.

PROBLEM XX.

Given the hight of an arc and the radius of the circle, to find the chord of half the arc.

RULE. Take the square root of twice the product of the radius and apothem; or, $k' = \sqrt{2\,r\,(r-a)} = \sqrt{2\,r\,h}$.

EXAMPLE. Find the chord of half the arc whose hight is 6 and radius 18.75.

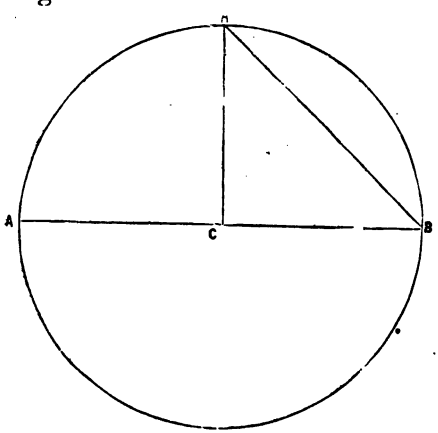


Fig. 23.

EXERCISE XV.

1. The chord B H, in half the arc A H B, in the above figure, is 20 feet and the chord A B,

of the whole arc 38 feet; what is the length of the arc?

Special Rule. From eight times the chord of half the arc subtract the chord of the whole arc; one-third of the remainder will give the arc, *nearly*.

2. The chord of an arc is 6 feet and the chord of half the arc is $3\frac{1}{2}$ feet; what is the length of the whole arc?

3. The chord of the whole arc is 40 and the height is 15; what is the length of the arc?

4. The radius of a circle is 30.8058 yards, the chord of an arc thereof is 36 yards; what is its apothem, and the chord of half the arc?

5. What is the chord of an arc whose height is 4, the radius of the circle being $54\frac{1}{4}$?

SECTOR, SEGMENT AND LUNE.

43. Let l = length of arc, k = chord, a = apothem, n = number of degrees it contains, h = height of segment, and A′ = area of lesser segment. Then, we have the following formulas:

SYNOPSIS OF FORMULAS.

44. For Sector. $A = \frac{1}{2} l r$ (XLIII): combining

this with XXXVIII we have $A = .008726\ n\ r^2$ (XLIV).

For Segment. Area $= A \pm A'$, according as the segment is greater or less than a semicircle (XLV); also,—

$$\text{Area} = \tfrac{2}{3}\,k\,h + \frac{h^3}{2\,k} \textit{ nearly}, \text{ (XLVI)}.$$

Special Rule. Divide the height of the segment by the diameter of the circle to three places of decimals. Find the quotient in the column *Height*, of the Table of Areas of the segments of a circle, at the end of this treatise, and take out the corresponding *Area Segment*, multiply this by the square of the diameter and the product will be the area of the segment required.

NOTE 1. If the quotient of the height by the diameter be greater than .5, subtract it from 1 and find the *Area Segment* corresponding to the remainder; subtract this from .7854 for the correct area segment.

NOTE 2. If the quotient of the height by the diameter does not terminate in three figures, find the *area segment* corresponding to the first three decimal figures of the quotient; subtract it from the next greater area segment; multiply the remainder by the fractional part of the quotient and add the product to the area segment first taken out of the table. When great accuracy is not required the fractional part may be omitted.

For Lune. Area $= A - A'$ (XLVII).

PROBLEM XXI.

Given the length of arc, and radius of a circle, to find the area of a sector.

RULE. Multiply the radius by half the length of the arc; or $A = \frac{1}{2} l r$.

EXAMPLE. Find the area of a sector of a circle whose radius is 90 feet, the arc of the sector being 80 feet in length?

SOLUTION. $A = \frac{1}{2} l r = \frac{1}{2} \times 80 \times 90$ feet $= 40 \times 90$ ft. $= 3600$ sq. ft.; 3600 sq. ft. $\div 9 = 400$ sq. yards, Ans.

PROBLEM XXII.

Given the number of degrees in the arc of a sector and the radius of a circle, to find the arc of the sector.

RULE. Multiply the number of degrees in the arc by the square of the radius and the product by the decimal .008726; or $A = .008726\, n\, r^2$.

EXAMPLE. Find the area of a sector whose arc contains 120°, the radius of the circle being 40 yards.

SOLUTION. $A = .008726\, n\, r^2 = .008726 \times 120 \times 40^2 = .008726 \times 120 \times 1600 = 1675.392$ square yards, Ans.

PROBLEM XXIII.

Given the length of the arc of a segment of a circle, its chord, or apothem, and the radius of the circle, to find the area of the segment.

RULE. Subtract the area of the associate segment from the area of the sector; or Area = A − A′.

EXAMPLE 1. Find the area of a segment whose arc contains 150°, the diameter of the circle being 60 feet, and the apothem 6 feet.

SOLUTION. Here, (formula XXXVIII) length of arc = .008726 × 150 × 60 = 78.534 feet. Area of sector = ½ l r = ½ × 78.534 × 30 = 1178.01 sq. ft. Area of triangle whose base is 58.787 (XXXIX), and altitude (apothem), 6, equals ½ × 58.787 × 6 = 177.36 sq. ft. Hence Area of Segment = A − A′ = 1178.01 − 176.36 = 1001.65 square feet, Ans.

EXAMPLE 2. What is area of a segment whose height is 2, and the diameter of the circle 20?

NOTE. *Do this by Special Rule.*

EXAMPLE 3. Find the area of a segment of a circle whose arc contains 240° and radius and apothem are 5 and 3 respectively?

PROBLEM XXIV.

Given data for finding greater and lesser segment, to find the area of a lune?

RULE. Subtract the area of the lesser segment from the area of the greater segment; or Area = A − A′.

EXAMPLE 1. Find the area of a lune, the outer arc containing 240° and the inner 32°, the radius of the smaller circle being 23 feet, and of the larger 80 feet, and the common chord being 38 feet.

SOLUTION. Here, (formula VIII), apothem of longer arc = $\sqrt{23^2 - 19^2}$ = 12.96, and apothem of shorter arc = $\sqrt{80^2 - 19^2}$ = 77.7; and length of outer arc = .008726 n d = 96.335, and of the inner arc = .00872 n d = 44.68: now the sector of the smaller circle = ½ l r = 1107.85, and the associate triangle = 246.24; therefore the greater segment = A + A′ = 1107.85 + 246.24 = 1354.09. Again, the sector of the greater circle = ½ l r = 1787.2, and the associate triangle = 1476.3; therefore, the lesser segment = A − A′ = 1787.2 − 1476.3 = 310.9, and finally, the area of the lune = A = A′ = 1354.09 − 310.9 = 1043.19 sq. feet, Ans.

EXERCISE XVI.

1. Let the chord be 40, the height of the

greater arc be 12, and the lesser 4; what is the area of the lune?

NOTE. If two lines in a circle cut each other, the product of the segments of the one equals the product of the segments of the other; hence may be found the diameters.

2. Let the chord be 40 and the heights of the segments be 15 and 2 respectively; what is the area of the lune?

LONG IRREGULAR FIGURES.

45. The contents of irregular figures or those not capable of being reduced to any known figure may be found by the method of equidistant ordinates.

RULE. Take several breadths at equal distances and divide the sum of the end measurements by two for the arithmetical mean between those two. Add together this mean and all the intermediate breadths and divide the sum by the number of addends; the quotient will give the mean breadth of the whole; this multiplied by the given length will give the area of the figure very nearly.

It is not necessary, sometimes, to take the equidistant breadths. Compute each trapesoid separately and the sum of them will be the area of the entire figure, *nearly*. Or, add all the

breadths together and divide by the number of these for a mean breadth, this multiplied by the length will give the area, *nearly*.

EXERCISE XVII.

1. Let the ordinate A B, be $10\frac{3}{8}$; a b, 7; c d, $8\frac{1}{2}$; e f, 12. and C D, $11\frac{1}{8}$; and the length A C, 15; what is the area?

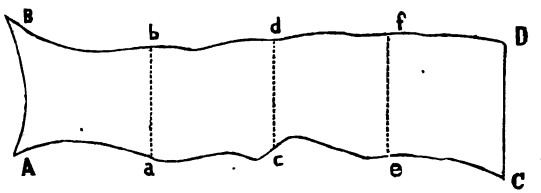


Fig. 24.

SOLUTION. $(10\frac{3}{8}+11\frac{1}{2})\div 2 = 10\frac{3}{4}$, the mean breadth of the ends; $(10\frac{3}{4}+7+8\frac{1}{2}+12)\div 4 = 9.5625$; then $9.5625\times 15 = 143.43\frac{3}{4}$, Ans.

2. The length of an irregular figure is 40 feet, its breadths in 6 equidistant places are 3, 7, 5, 9, 8 and 11 inches, the first and last being the end measures: required the area.

3. The length of an irregular figure or plot of ground is 100 yards, and its breadths at five equidistant places are 10, 14, 11, 23, and 26 yds.; what is the area?

4. The length of an irregular figure is 36.6

and the breadths at nine equidistant places are 0, 4.4, 6.5, 7.6, 5.4, 8, 5.2, 6.5, 6.1; required the area.

EQUIVALENT AREAS.

PROBLEM XXV.

Given the diameter of a circle, to find the side of a square equal in area to the circle.

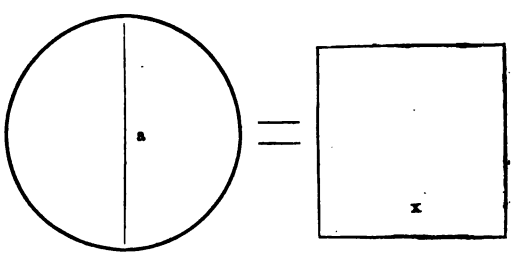


Fig. 25.

NOTE. In this and the following seven problems, a = the part given and x = the part required. The sign = should be read *is equivalent to.*

RULE. Multiply the diameter by .8862269; or $x = a \times .8862269$ (XLVIII).

PROBLEM XXVI.

Given the circumference of a circle to find

the side of a square equal in area to the circle.

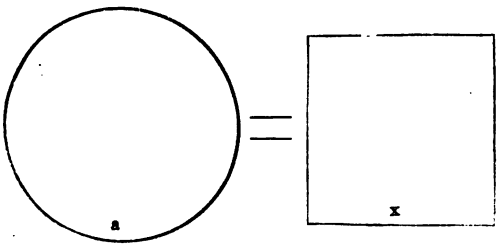


Fig. 26.

RULE. Multiply the circumference by .2820948; or, $x = a \times .2820948$, (XLIX).

PROBLEM XXVII.

Given the diameter of a circle, to find the side of an inscribed square. See fig. 27.

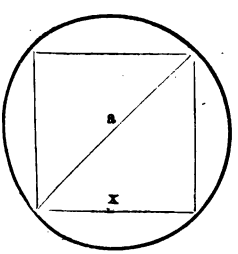


Fig. 27.

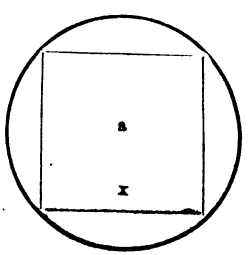


Fig. 28.

RULE. Multiply the diameter by .7071068; or $x = a \times .7071068$, (L).

PROBLEM XXVIII.

Given the area of a circle, to find the side of the inscribed square. See fig. 28.

RULE. Multiply the area by .6366297, and extract the square root of the product; or $x = \sqrt{a \times .6366197}$, (LI).

PROBLEM XXIX.

Given the side of a square to find the diameter of the circumscribed circle. See fig. 29.

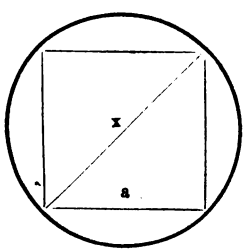


Fig. 29,

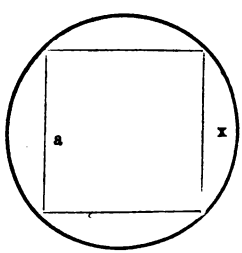


Fig. 30.

RULE. Multiply the side of the square by 1.4142136; or $x = a \times 1.4142136$ (LII).

PROBLEM XXX.

Given the side of a square to find the circumserence of the circumscribed circle. See fig. 30.

RULE. Multiply the side of the square by 4.4428934; or $x = a \times 4.4428934$, (LIII).

PROBLEM XXXI.

Given the side of a square, to find the diameter of a circle equal in area.

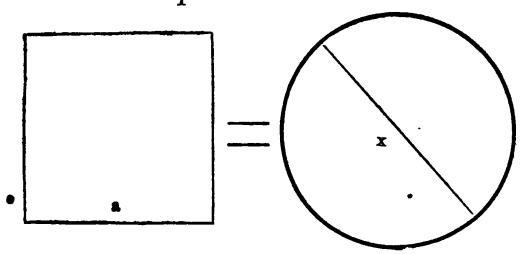


Fig. 31.

RULE. Multiply the side of the square by 1.1283791 ; or $x = a \times 1.1283791$, (LIV).

PROBLEM XXXII.

Given the side of a square, to find the circumference of a circle equal in area.

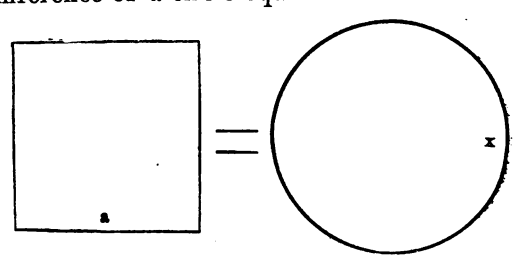


Fig. 32.

RULE. Multiply the side of the square by 3.5449076; or $x = a \times 3.5449076$, (LV).

EXERCISE XVIII.

**Involving the last eight Problems.*

1. The diameter of a circle is 30, what is the side of a square equal in area?

2. The circumference of a circle being 80 yards, what is the side of a square equal in area?

3. The diameter of a circle is 20, what is the length of the side of the inscribed square?

4. The area of a circle is 10 sq. inches: required the side of the inscribed square.

5. If the side of a square be 10 feet, what will be the diameter of the circumscribed circle?

6. If the side of a square is 10 feet, what is the circumference of the circumscribed circle?

7. If the side of a square be 100, what is the diameter of a circle equal in area?

8. The side of a square is 10 inches, what is the circumference of a circle equal in area?

ELLIPSE.

46. If a plane be passed through a right cone making a less angle with the base than does the

*It will be sufficient to carry the decimal to the third place, observing to add one to the last figure in the decimal if the rejected quantity be equal or greater than one-half.

side of the cone, the surface thus made is called an Ellipse.

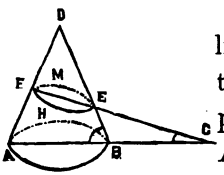


Fig. 33.

Here F M E H, is an Ellipse, since the angle which the plane of it makes with the plane of the base of the cone, A H B, is less than the angle D B A, which the side of the cone makes with the base A H B.

☞See page for definition of cone and right cone.

An ellipse may be conveniently described thus: Assume any two points as f and f′, and set down two pins; fasten together the ends of a string of suitable length and through it over these pins: with a pencil holding the string out with equal tension, trace an ellipse.

NOTE. The figure will become more elongated as the string is shortened.

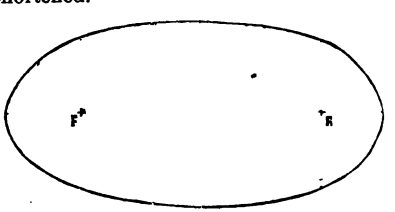


Fig. 34.

47. The points f and f′ are called the **foci** of the ellipse.

48. The line passing through the foci and terminating in the circumference is called the transverse diameter, or axis. See fig. 35.

49. The line bisecting the transverse axis at right-angles and terminating in the circumference is called the conjugate diameter, or axis.

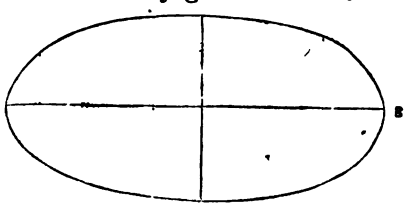


Fig. 35.

50. If from any point in the circumference, as P, two straight lines be drawn to the foci, the sum of these lines will be equal to that of the two lines drawn from any other point as P′; or, in other words, it will be always equal to the transverse diameter.

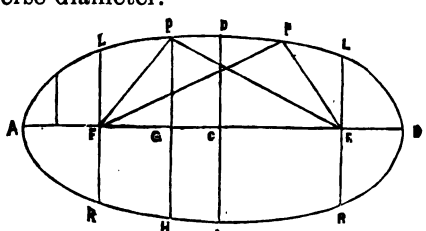


Fig. 36.

Further let it be remarked as to fig. 36, that

the point O in the middle of the axis A B and C D, is the centre of the ellipse: the line L R, drawn through the focal point of f or f′ perpendicular to the transverse diameter A B, is called the parameter. A line drawn from any point of the curve, perpendicular to the transverse diameter is called an ordinate to that diameter, or axis, as P G or H G. When it goes quite through the ellipse, as P H, it is called a double ordinate. The extremity of any diameter is called the vertex; thus, A and B are the vertices of the diameter A B, and C and D are the vertices of the diameter C D. That part of the diameter between the vertex and the ordinate is called an abscissa; thus, A G and G B are the abscissas to the ordinate P G.

51. In the ellipse let c = circumference, t = transverse axis, k = conjugate axis, a = abscissa, o = ordinate, A = area, and d = distance from centre of transverse axis to the the foot of ordinate: then we shall have the following:

SYNOPSIS OF FORMULAS.

$$c = \pi\sqrt{\frac{t^2+k^2}{2}}\ \text{(LVI)};\quad t = \frac{k\,a}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k^2}{4} - o^2}\right\}$$

(LVII); add if less abscissa is given. $k = \frac{o\,t}{\sqrt{(t-a)\,a}}$

(LVIII); $o = \frac{k}{t}\sqrt{(t-a)a}$ (LIX); $a = \frac{t}{2} \pm d$, d being equal to $\frac{t}{k}\sqrt{(\frac{k}{2}+o)(\frac{k}{2}-o)}$; (LX) $A = \frac{\pi t k}{4} = .7854\ t\ k$ (LXI).

PROBLEM XXXIII.

Given the transverse and conjugate diameters of an ellipse, to find the circumference.

RULE. Multiply the square root of half the sum of the squares of the transverse and conjugate diameters by 3.1416; or $c = \pi\sqrt{\frac{t^2+k^2}{2}}$.

EXAMPLE. In an ellipse the transverse and conjugate axis are 24 and 20 inches respectively; what is the circumference?

Applying the formula we have $c = \pi\sqrt{\frac{t^2+k^2}{2}}$ $= 3.1416\sqrt{\frac{24^2+20^2}{2}} = 3.1416\ \sqrt{488} = 3.1416 \times 22.04+ = 69.24+$ inches, Ans.

PROBLEM XXXIV.

Given abscissa, ordinate, and conjugate diameter, to find the transverse diameter.

RULE. Add together half the conjugate diameter and the square root of the difference of a fourth of the square of the conjugate diameter and the square of the ordinate. Mul-

tiply this by the product of the conjugate diameter and the abscissa divided by the square of the ordinate: or $t = \frac{k\,a}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k}{4} - o^2}\right.$.

NOTE. In applying this rule the smaller abscissa must be used.

EXAMPLE. Find the transverse axis of an ellipse whose conjugate axis is 15, an ordinate 6 and smaller abscissa, 9.

Applying the formula we have—

$$t = \frac{k\,a}{o^2}\left\{\frac{k}{2} + \sqrt{\frac{k^2}{4} - o^2}\right\} = \frac{15 \times 9}{6^2}\left\{\frac{15}{2} + \sqrt{\frac{15^2}{4} - 6^2}\right.$$

$$= \frac{15}{4} \times 12 = 45, \text{ Ans.}$$

PROBLEM XXXV.

Given abscissa, ordinate, and transverse axis, to find the conjugate axis.

RULE. Divide the product of the ordinate and transverse axis by the square root of the product of the abscissa into the difference of the transverse axis and abscissa: or $k = \frac{o\,t}{\sqrt{(t-a)\,a}}$.

EXAMPLE. Find the conjugate axis of an ellipse whose transverse axis is 45, ordinate 6, and abscissa 9.

Here $k = \frac{o\,t}{\sqrt{(t-a)\,a}} = \frac{6 \times 45}{\sqrt{(45-9)\,9}} = \frac{6 \times 45}{18} = 45$, Ans.

PROBLEM XXXVI.

Given the two axes and an abscissa, to find the ordinate.

RULE. Multiply the quotient arising from dividing the conjugate axis by the transverse axis, by the square root of the product of the abscissa into the difference of the transverse axis and abscissa: or $o = \frac{k}{t}\sqrt{(t-a)\,a}$.

PROBLEM XXXVII.

Given the two axes and an ordinate to find the abscissa.

RULE. Add the distance from center of transverse axis to the foot of the ordinate, to half the transverse axis, if the greater abscissa be required, and subtract it, if the smaller be required: or $a = \frac{t}{2} \pm d$. d being equal to $\frac{t}{k}\sqrt{(\frac{k}{2}+o)\,(\frac{k}{2}-o)}$.

EXAMPLE. Find the abscissa of an ellipse whose axes are 45 and 15, and ordinate 6.

Here $d=\frac{t}{k}\sqrt{(\frac{k}{2}+o)\,(\frac{k}{2}-o)} = \frac{45}{15}\sqrt{(\frac{15}{2}+6)(\frac{15}{2}-6)} = 3\times\frac{1}{2}=12\frac{1}{2}$. Then, $a = \frac{t}{2} + d = 22\frac{1}{2}+13\frac{1}{2} = 36$, the greater abscissa; and $a = \frac{t}{2} - d = 22\frac{1}{2} - 13\frac{1}{2} = 9$, the lesser abscissa.

PROBLEM XXXVIII.

Given the two axes of an ellipse, to find its area.

RULE. Multiply the product of the axes by the decimal .7854; or A = .7854 t k.

EXAMPLE. Find the area of an ellipse whose diameters are 16 and 9.

Here A=.7854 t k = .7854×16×9=113.1, Ans.

EXERCISE XIX.

Involving the last six problems.

1. The transverse axis of an ellipse is 30, the conjugate axis 20; what is the circumference?

2. The longer diameter of an ellipse is 70, and the shorter 50; what is the area?

3. The transverse and conjugate diameters of an ellipse are 60 and 40, and of another concentric with it, 80 and 10; required the area of the space between the circumferences.

4. A gentleman has an elliptical flower garden whose greater diameter is 30, and lesser, 24 feet; and he has ordered a gravel walk to be made round it of 5 feet 6 inches in width; what is the area of the walk?

5. What is the circumference of an ellipse whose diameters are 7 and 5?

6. Let the transverse axis be 35 and the conjugate axis 25, and abscissa 7: required the ordinate.

7. Let the diameters be 35 and 25 and the ordinate 10: required the abscissa.

8. Let the transverse axis be 35, the ordinate 10 and the abscissas 28 and 7: required the conjugate axis.

9. Let the transverse axis be 120, the ordinate 16 and the abscissas 25 and 96: required the conjugate axis.

10. What is the transverse axis of an ellipse whose conjugate axis is 70, an ordinate 28, and one abscissa 168?

PARABOLA.

52. If a plane be passed through a right cone parallel to its side, the figure thus generated

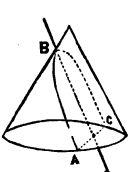


Fig. 37.

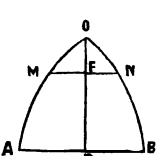


Fig. 38.

is called a parabola. Fig. 37 represents a parabola.

53. Any double ordinate, as A B, in figure 38, may be called the base of a parabola, and the abscissa O D, to that ordinate is its hight.

54. The parameter of a parabola is the double ordinate passing through its focus: *it is a third proportional to the abscissa and ordinate of any point in the curve.* Let p represent parameter M N of the parabola, fig. 38: then, abscissa O D : ordinate D B :: ordinate D B : p. Hence the ordinate and abscissa being given the parameter is readily found.

55. The *focus* F, is always *one fourth of the parameter distant from the vertex* O.

56. The parabola may be defined as a curve, such that each of its points is at the same distance from a given point and a given straight line. Thus in figure 39, let B C be the given line (*it is called the directrix*), and let F, the *focus*, be the given point. It will be seen from the above definition that the curve may be generated by a continuous movement. Thus, place one side, D C, of a right-angled triangular ruler, D C E, against the directrix; fasten one end of a string equal in length

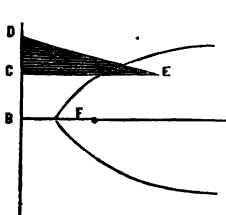


Fig. 39.

to the side C E, at the point E, and the other end at the *focus* F; press a pencil against the string and ruler, and as the ruler is moved along the directrix B C, the point of the pencil will describe a parabola.

57. In the solution of problems of the parabola, let p = parameter, a = any abscissa, o = its corresponding ordinate, b = base or double ordinate, A = area, and l = parabolic curve: then we shall have,—

SYNOPSIS OF FORMULAS.

$A = \frac{2}{3}$ ab (LXII); $l = 2\sqrt{o^2 + \frac{4}{3}a^2}$. (LXIII); $p = \frac{o^2}{a}$ (LXIV).

PROBLEM XXXIX.

Given the abscissa and double ordinate, or base of a parabola, to find its area.

RULE. Multiply two-thirds of the abscissa by the base; or $A = \frac{2}{3}$ ab.

EXAMPLE. Find the area of a parabola whose base is 15 and hight 11.

Here, by formula LXII, $A = \frac{2}{3}$ ab $= \frac{2}{3} \times 15 \times 11 = 110$, Ans.

PROBLEM XL.

Given the ordinate and abscissa of a parabola to find the parabolic curve, that is, the length of A O B, figure 38.

RULE. Multiply the square root of the sum of the square of the ordinate and four-thirds the square of the abscissa by two; or $l = 2\sqrt{o^2 \times \frac{4}{3}a^2}$.

EXAMPLE. Find the length of the parabolic curve, of which the ordinate and abscissa are respectively 15 and 4.

Here, formula LXIII, $l = 2\sqrt{o^2 + \frac{4}{3}a^2} = 2\sqrt{15^2 + \frac{4}{3} \times 4^2} = 2\sqrt{225 + 21.33} = 2 \times 15.69 = 31.38$, Ans.

PROBLEM XLI.

Given the ordinate and abscissa of a parabola, to find the parameter.

RULE. Divide the square of the ordinate by the abscissa; or $p = \frac{o^2}{a}$.

EXAMPLE. Find the parameter of a parabola whose ordinate is 10 and abscissa $16\frac{2}{3}$.

Here, formula LXIV, $p = \frac{o^2}{a} = \frac{10^2}{16\frac{2}{3}} = \frac{100}{16\frac{2}{3}} = 6$, Ans.

EXERCISE XX.

Involving the last three Problems.

1. What is the area of a parabola, whose base is 24 and hight 4?

2. What is the length of the curve of a parabola, whose abscissa is 3, and ordinate 8.

3. What is the length of the curve of a parabola, cut off by a double ordinate whose length is 12, the abscissa being 2?

4. Find the area of a parabola whose base is 75, and hight 48 chains.

5. Given an ordinate of a parabola 60, and its abscissa 42: required the parameter.

HYPERBOLA.

58. A hyperbola may be cut from a right cone, by a plane which makes with the base an angle greater than that made by the side of the cone.

59. The hyperbola may be defined without reference to the cone from which it may be cut, as a plane curve, such that the difference of the distances from any point of it to two fixed points is equal to a given distance.

The fixed points are called *foci*, and the given distance is that portion of the straight line drawn through the *foci*, which is included between the two branches of the curve.

It may be seen by the accompanying figure how the points, curves and line may be developed. Here a double cone is used, the plane being passed through both of them, thus generating two hyperbolic curves having a mutual dependence upon each other in regard to their elements.

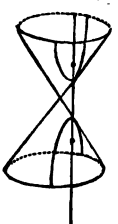


Fig. 40.

Either of these curves may be constructed by a continuous movement, as follows:

In figure 41, let f and f′ be the *foci* of the two associate curves of a hyperbola: fasten one end of a ruler f′ C, of sufficient length, at f′, and let it be free to revolve about it: attach a string at C, whose length shall be less than the length of the ruler by any given distance (less than f f′): fasten the other end of the string at f: press a pencil P against the string, keeping it close to the ruler, and keeping the string tense;

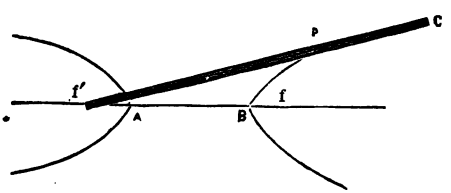


Fig. 41.

then move the ruler about f′: the pencil will

describe one branch of the curve: a similar construction with f as the center will give the other branch.

61. In this construction the difference of the distances of the point P from the *foci* f and f′ will always be equal to P f′ − P f = A B.

62. In figure 42, f and f′ are the *foci*, A B is the transverse axis, C is the center

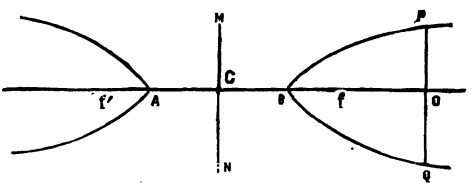


Fig. 42.

M N is the conjugate axis, the points M and N being distant from A or B by C f or C f′: P O is the ordinate, and P Q the double ordinate, or base of the area P B Q; O B is the smaller abscissa and A O the greater abscissa.

63. For the solution of problems of the hyperbola, let the symbols be the same as in the ellipse and parabola: then we shall have the following,—

SYNOPSIS OF FORMULAS.

$$A = \frac{4\,k\,a}{75\,t}\left\{3\ \sqrt{7\,a\,(7\,t+5a)} + 4\ \sqrt{a\,t}\right\}\ \text{(LXV)};$$

$t = \frac{k\,a}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k^2}{4} + o^2}\right\}$ (LXVI); + or − according as the smaller or greater abscissa is given.

$k = \frac{o\,t}{\sqrt{(t+a)\,a}}$ (LXVII); $a = d \pm \frac{t}{2}$ where $d = \frac{t}{k}\sqrt{\frac{k^2}{4} + o^2}$ (LXVIII); $o = \frac{k}{t}\sqrt{(t+a)\,a}$ (LXIX).

PROBLEM XLII.

Given the transverse and conjugate axes, and the abscissa of an hyperbola, to find the area cut off by the double ordinate.

EXAMPLE. Find the area of an hyperbola whose axes are 60 and 36, and lesser abscissa 20.

SOLUTION. By formula (LXV), we have $A = \frac{4\,k\,a}{75\,t}\left\{3\sqrt{7\,a\,(7\,t+5\,a)} + 4\sqrt{t\,a}\right\} = \frac{4 \times 36 \times 20}{75 \times 60}$ $\left\{3\sqrt{7 \times 20\,(7 \times 60 + 5 \times 20)} + 4\sqrt{60 \times 20}\right\} = \frac{16}{25}$ $\{3\sqrt{72800} + 4\sqrt{1200} = \frac{16}{25} \times (809.4 + 138.56) = \frac{16}{25} \times 947.96 = 606.77+$, Ans.

PROBLEM XLIII.

Given the conjugate axis, abscissa and ordinate of an hyperbola to find the transverse axis.

EXAMPLE. Find the transverse axis of a hyperbola whose ordinate is 20, smaller abscissa $10\frac{2}{3}$, and conjugate axis 30.

SOLUTION. By formula LXVI, we have, $t = \frac{k\,a}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k^2}{4} + o^2}\right\} = \frac{30 \times 10\frac{2}{3}}{400}\left\{\frac{30}{2}\sqrt{\frac{900}{4} + 400}\right\} = \frac{4}{5}$ $(15 + 25) = \frac{4}{5} \times 40 = 36$, Ans.

PROBLEM XLIV.

Given the transverse axis, ordinate and abscissa of an hyperbola, to find the conjugate axis.

EXAMPLE. The transverse axis of an hyperbola is 60, the ordinate 24, and the lesser abscissa 20: required the conjugate axis.

SOLUTION. By formula LXVII, $k = \frac{o\,t}{\sqrt{(t + a)\,a}} = \frac{24 \times 60}{\sqrt{80 + 20}} = \frac{1440}{\sqrt{1600}} = 36$, Ans.

PROBLEM XLV.

Given the two axes and an ordinate of a hyperbola, to find the abscissas.

EXAMPLE. The axes of a hyperbola are 120 and 72, and the ordinate 48: required the abscissas.

SOLUTION. By formula, $a = d \pm \frac{t}{2}$ Now, $d = \frac{t}{k}\sqrt{\frac{k^2}{4} + o^2} = \frac{120}{72}\sqrt{1296 + 2304} = 100$, and substituting the value of d, we have, $a = 100 \pm \frac{120}{2} = 100 + 60 = 160$ or 40, Ans.

PROBLEM XLVI.

Given the two axes and an abscissa of a hyperbola, to find the ordinate.

NOTE. The smaller abscissa is implied. The greater is always found by adding the smaller to the transverse axis.

EXAMPLE. What is the ordinate of a hyperbola whose axes are 30 and 15, and smaller abscissa 10?

SOLUTION. By formula LXIX, $o = \frac{k}{t}\sqrt{(t+a)\,a} = \frac{15}{30}\sqrt{(30+10)\,10} = \frac{1}{2}\sqrt{400} = \frac{1}{2}$ of $20 = 10$, Ans.

MENSURATION OF SOLIDS.

64. A **solid** is that which has length, breadth and thickness.

65. The **solid contents** of any body is the number of cubic inches, feet, yards, etc., it contains.

66. A **cube** is a solid having six equal sides at right angles to one another. Fig. 43.

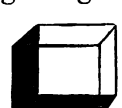


Fig. 43.

Fig. 44

67. A **prism** is a solid whose ends are plane figures, which are parallel, equal and similar. Fig. 44.

NOTE. Figures which are similar are those which are mutually equiangular, and which have the sides about the equal angles, taken in the same order, proportional.

68. A prism is called *triangular*, when its ends are triangles; a *square prism*, when its ends are square; *pentagonal*, when its ends are pentagons.

69. A **parallelopiped** is a solid having rectangular sides, each oposite pair being equal and parallel. Fig. 45.

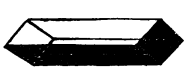


Fig. 45.

Fig. 46.

70. A **cylinder** is a round solid, having circular ends, and may be conceived to be described by the revolution of a rectangle about one of its sides, which remains fixed. Fig. 46.

71. A **pyramid** is a solid having a plane figure for its base, and whose sides are triangles, having angles meeting at a point called the vertex, Fig. 47. Pyramids like prisms take their names from their bases. When the base is a triangle, the solid is called a *triangular pyra-*

mid; when the base is a square, it is called a *square pyramid*, etc.

72. A **cone** is a round pyramid having a circle for its base. Fig. 48.

Fig. 47.

Fig. 48.

Fig. 49.

73. A **sphere** is a round solid which may be conceived to be formed by the revolution of a semicircle about the diameter which remains fixed. Fig. 49.

74. The axis of a solid is a line joining the middle of both ends.

75. When the axis is perpendicular to the base, the solid is called a right prism, pyramid, cone, etc., as the case may be. Otherwise, they are respectively oblique.

76. The hight or *altitude* of a solid is a line drawn from its vertex perpendicular to its base, and is equal to the axis of a right prism, pyramid or cone; but in an oblique figure the altitude is the perpendicular of a right-angled triangle whose hypothenuse is the axis.

77. When the base is a regular figure, the solids are called regular; as a regular prism,

regular cone, etc. But when the base is irregular, the solid is called irregular; as an irregular prism, cone, etc.

78. The **segment** of any solid is a part cut off from the top, by a plane parallel to its base.

79. A **frustum** is the part left at the bottom, after the segment is cut off.

80. A **wedge** is a solid having a rectangular base and two opposite sides meeting in an edge.

81. A **spherical segment** is a part of a sphere cut off by a plane.

82. A **spherical zone** is a part of a sphere intercepted between two plains which are parallel to each other.

83. A **spheroid** is a solid generated by the rotation of a semi-ellipse about one of its axes, which remains fixed.

84. A **paraboloid** is a solid generated by the revolution of a semi-parabola about its axis, which remains fixed.

85. An **hyperboloid** is a solid generated by the revolution of a semi-hyperbola about its axis, which remains fixed.

86. A **regular body** is a solid contained by a certain number of similar and equal planes;

these planes are called the sides of the body or *polydron*, as it is sometimes called, the lines bounding its sides are called its edges.

Only five *regular bodies*, or regular *polyhedrons* can possibly be found. Because it is proved in solid geometry that only three kinds of equilateral and equiangular plane figures joined together can make a solid.

87. A **tetrahedron** is a regular solid contained by four equilateral triangles.

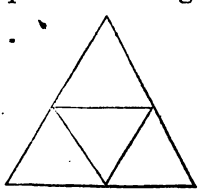


Fig. 50.

88. A **hexahedron** is a regular solid contained by six square faces or sides.

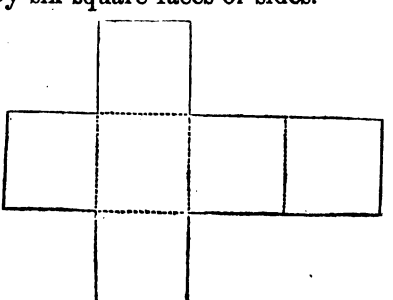


Fig. 51.

89. An **octahedron** is a regular solid contained by eight equilateral triangles.

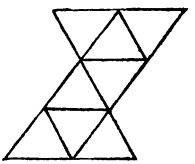


Fig. 52.

90. A **dodecahedron** is a regular solid contained by twelve regular pentagons.

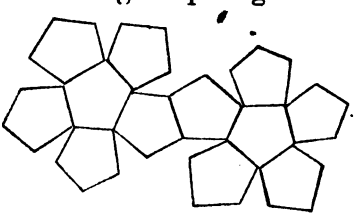


Fig. 53.

91. An **icosahedron** is a regular solid contained by twelve equilateral triangles.

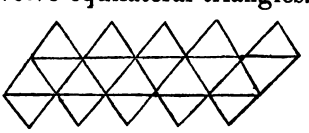


Fig. 54.

NOTE. If figures similar to those given above be drawn on card-board, and cut out, by cutting through the outside boundary lines and but partially cutting the inner divisions, and then folded up, the bodies defined will be formed.

MEASUREMENT OF REGULAR SOLIDS.

92. Let s = surface, v = volume, or contents, and let e = edge: then we have the following

SYNOPSIS OF FORMULAS.

For Tetrahedron. $s = e^2 \sqrt{3} = 1.732\ e^2$ (LXX);
$v = \frac{1}{12} e^3 \sqrt{2} = .11785\ e^3$ (LXXI).

For Hexahedron. $s = 6\ e^2$ (LXXII): $v = e^3$ (LXXIII).

For Octahedron. $s = 2\ e^2 \sqrt{3} = 3.464\ e^2$ (LXXIV);
$v = \frac{1}{3} e^3 \sqrt{2} = .4714045\ e^3$ (LXXV).

For Dodecahedron $s = 15\ e^2 \sqrt{\frac{1}{3}(5 + 2\sqrt{5})} =$
$20.645775\ e^2$ (LXXVI);

$$v = 5\ e^3 \sqrt{\frac{45 + 21\sqrt{5}}{40}} =$$

$7.6631\ e^3$ (LXXVII).

For Icosahedron. $s = 5\ e^2 \sqrt{3} = 8.66\ e^2$ (LXXVIII);
$v = \frac{5}{12} e^3 \sqrt{\frac{1}{2}(7 + 3\sqrt{5})} =$
$2.18169\ e^3$ (LXXIX).

EXAMPLES.

1. Find the surface and volume of a tetrahedron whose edge is 20 inches.

SOLUTION. By formula LXX, $s = e^2 \sqrt{3} = 1.732\ e^2 = 1.732 \times 20^2 = 692.8$ sq. in., Ans. $v = .11785\ e^3$ (LXXI) $= .11785 \times 20^3 = 942.8$ cu. in., Ans.

2. Find the surface and solid contents of a hexahedron or cube, whose edge is 9 inches.

SOLUTION. By formula LXXII, $s = 6\ e^2 = 6 \times 9^2$

$= 486$ sq. in. By formula LXXIII, $v = e^3 = 9^3 = 729$ cu. in., Ans.

3. Find the surface and solid contents of an octahedron, whose edge is 2 feet.

SOLUTION. By formula LXXIV, $s = 3.464\, e^2 = 3.464 \times 2^2 = 13.856$ sq. ft. By LXXV, $v = .4714045 \times 2^3 = 3.771236$ cu. ft., Ans.

4. Find the surface and volume of a dodecahedron, whose edge is 4 feet 2 inches.

SOLUTION. By formula LXXVI, $s = 20.645775\, e^2 = 20.645775 \times (50 \text{ in.})^2 = 358.433$ sq. ft. By LXXVII, $v = 7.6631\, e^3 = 7.6631 \times (50 \text{ in.})^3 = 554.349$ cu. ft., Ans.

5. Find the surface and volume of an icosahedron, whose edge is 3 inches.

SOLUTION. By formula LXXVIII, $s = 8.66\, e^2 = 8.66 \times 3^2 = 77.94$ sq. in. By LXXIX, $v = 2.18169\, e^3 = 2.18169 \times 3^3 = 58.90463$ cu. in., Ans.

TABLE II.

SHOWING THE SOLIDITY OF THE FIVE REGULAR BODIES THE LENGTH OF A SIDE IN EACH BEING ONE.

No. of Sides.	Names.	Solidity.
4	Tetrahedron	.1178511
6	Hexahedron	1.000000
8	Octahedron	.4714045
12	Dodecahedron	7.6631189
20	Icosahedron	2.181695

92. Now, as regular solids are to each other as the *cubes* of their like dimensions, it follows that the solid contents of any regular figure may be obtained by multiplying the cube of its edge by the number representing the solidity set opposite its name in the above table.

NOTE. This method may be used in proof of the process by the formulas.

GENERAL PROBLEMS IN THE MENSURATION OF THE FIVE REGULAR BODIES.

1. Find the surface and contents of an icosahedron whose edge is 4.

2. Fine the surface and solidity of a cube, or hexahedron whose edge is 20.

3. Find the surface and solid contents of a tetrahedron whose edge is 8.

4. Find the surface and solid contents of a dodecahedron whose edge is 10.

5. Find the surface and volume of an octahedron whose edge is 11.

6. Find the surface and volume of an icosahedron whose edge is 5 yards.

RIGHT AND OBLIQUE PARALLELOPIPEDS, PRISMS, AND CYLINDERS.

93. Let a = area of base, or *end*, p = perimeter of base and p′ = the perimeter of section per-

pendicular to one of the edges of the solid; also, let h = hight, s = the whole surface, and v = volume, or solid contents. Then we have the following:

SYNOPSIS OF FORMULAS.

$v = a\,h$ (LXXX); $s = h\,p + 2\,a$ (LXXXI), when the solid is *right*, $s = h'\,p + 2\,a$, or $s = h\,p' + 2\,a$ (LXXXII), when the solid is oblique. h', represents the perpendicular, as not falling through the center of the solid.

EXAMPLES.

1. What are the surface and solid contents of a prism whose hight is 20 inches, and base an equilateral triangle four inches on a side.

SOLUTION. By formula (v) $a = .433\,b^2 = 433 \times 4^2 = 6.928$ sq. inches = area of base; the perimeter $= 4 \times 3 = 12$. Hence, by formula (LXXXI), $s = h\,p + 2\,a = 20 \times 12 + 2 \times 6.928 = 253.856$ sq. in. = surface. And, by formula (LXXX), $v = a\,h = 6.928 \times 20 = 138.56$ cu. inches.

2. Find the surface of an oblique prism whose base is a regular hexagon, with edge equal to 10 inches, the lateral edges of the prism being each 40 feet long, and the perimeter of a section perpendicular to them $4\frac{1}{2}$ feet.

SOLUTION. By formula XXII, $A = \frac{1}{2}\,a\,n\,s = \frac{1}{2} \times 8.66025 \times 6 \times 10 = 259.8075$ sq. inches $= 1.8042$

sq. fee t= area of base. Hence, LXXXII, $s = h\,p' + 2\,a = 40 \times 4\frac{1}{2} + 2 \times 1.8042 = 180 + 3.6084 =$ 183.6084 sq. feet, Ans.

3. Find the surface and solidity of a right cylinder whose hight is 20 inches, and diameter 12 inches.

SOLUTION. By formula XXX, area of base = $A = \pi r^2 = 3.1416 \times 6^2 = 113.0976$ sq. in., And, $p = 3.1416 \times 12 = 37.6992$ inches. Hence, LXXXI, $s = h\,p + 2\,a = 20 \times 37.6992 + 2 \times 113.0976 =$ 980.1732 sq. in.; and, LXXX, $v = a\,h = 113.0976 \times 20 =$ 2251.952 cubic inches, Ans.

4. How many gallons of water will a cylindrical cistern hold, whose diameter is 6 feet, and depth 7 feet?

SOLUTION. By formula XXX, area of base = $A = \pi r^2 = 3.1416 \times 3^2 = 28.2744$ square feet. Hence, LXXX, $v = a\,h = 28.2744 \times 7 = 197.9208$ cubic feet. Now, as there are 231 cubic inches in a gallon, (see table at end of this work) and the number of gallons in a cubic foot cannot, hence, be exactly expressed, we reduce 197.9208 cubic feet to cubic inches, thus:—$197.9208 \times 1728 = 342007.1424$ cubic inches. Then,— $342007.1424 \div 231 = 1480.5504$ gallons, Ans.

EXERCISE XXI.

Involving Mensuration of Right and Oblique Parallelopipeds, Prisms and Cylinders.

1. Find the surface and cubic contents of a rectangular parallelopiped whose hight is 25 inches, its base being 4 inches wide and 5 inches long.

2. How many gallons of water are contained in a circular cistern whose diameter is 12 feet and depth 10 feet?

NOTE. This answer can be converted into barrels by dividing by 31½, or, into hogsheads by dividing by 63.

3. What is the surface and solid contents of a right prism whose base is an octagon, each side of which is 2 feet, the edges of the prism being each 18 feet long?

4. How many pails of water are contained in a pentagonal cistern whose depth is 15 feet, each edge of the bottom being 6 feet?

NOTE. A pail holds 10 quarts.

5. What is the surface and volume of an oblique prism, each end being a regular nonagon whose side is 20 inches, the edges of the prism being 14 feet, and the perimeter of a section perpendicular to them being 13 feet?

6. What is the surface and volume of a right

cylinder whose hight is 42 feet and circumference 22 inches?

REGULAR PYRAMID AND CONE.

94. Let p = perimeter of base, l = length of slant hight, h = hight of vertex above base, and a = area of base. Then we have the following:

SYNOPSIS OF FORMULAS.

$v = \frac{1}{3} a h$ (LXXXIII); $s = \frac{1}{2} p l + a$ (LXXXIV).

EXAMPLES.

1. Find the surface and volume of a regular cone whose slant hight is 20 inches and diameter of base, 10 inches.

SOLUTION.

By formula VI, $h = \sqrt{20^2 - 5^2} = \sqrt{400 - 25} = \sqrt{375} = 19.3649$.

" " XXV, $c = \pi d = 3.1416 \times 10 = 31.416$.

" " XXX, $a = \pi r^2 = 3.1416 \times 5^2 = 3.1416 \times 25 = 78.54$.

" " LXXXIII, $v = \frac{1}{3} a h = \frac{1}{3} \times 78.54 \times 19.3649 =$ 1520.9191 cubic inches.

" " LXXXIV, $s = \frac{1}{2} p l + a = \frac{1}{2} \times 31.416 \times 20 +$ 78.54 = 392.7 square inches, Ans.

2. What are the volume and surface of a right pyramid whose slant hight is 24 yards, the base being a regular pentagon whose side is 10 yards?

SOLUTION.

By formula XXII, $a = \frac{1}{2} a n s = \frac{1}{2} \times .688191 \times 5 \times 10 =$ 17.204. Also, perimeter $= 5 \times 10 =$ 50 yards.

" " VII, $h = \sqrt{24^2 - 6.88^2} = 22.99$.

NOTE. For the quantity 6.88 see Sec. 22.

" " LXXXIII, $v = \frac{1}{3} a h = \frac{1}{3} \times 17.204 \times 22.99 =$ 131.839 cubic yards.

" " LXXXIV, $s = \frac{1}{2} p l + a = \frac{1}{2} \times 50 \times 24 + 17.204$ $= 617.204$ square yards, Ans.

EXERCISE XXII.

Involving the Mensuration of Regular Pyramids and Cones.

1. Find the volume and surface of a right cone whose slant hight is 10 yards and diameter of base is 6 yards.

2. Find the surface and solid contents of a regular square pyramid whose slant hight is 20 inches, and area of base 576 square inches?

3. What is the surface and volume of a right cone whose base has a diameter of 14 feet, and whose hight is 10 feet?

4. Find the surface and solidity of a right pyramid whose hight is 18 feet, its base being a regular hexagon whose side is 9 feet?

5. The spire of a church, of a conical form measures 37.6992 feet round its base; what is

the solidity, its perpendicular hight being 100 feet?

6. How many cubic yards in an upright cone, the circumference of the base being 70 feet, and the slant hight 30 feet?

7. How many cubic feet in an oblique cone, the greatest slant hight being 20 feet, the least 16, and the diameter of the base 8 feet?

8. What is the convex surface and whole surface of a right pyramid, whose slant hight is 225 yards, and the base 640 square yards?

FRUSTUM OF PYRAMID OR CONE.

95. Let a and a' = areas of ends, h = hight, e and e' = edges of ends of frustum, p and p' = perimeters of ends; and, in case of cone, d and d' = diameters of ends, $\sqrt{d\,d'}$ = the mean proportional between these two diameters, and $\sqrt{a\,a'}$ = the mean proportional between the areas of the two ends. Then we have the following:

SYNOPSIS OF FORMULAS.

$v = \frac{1}{3} h\,(a + a' + \sqrt{a\,a'})$ (LXXXV);

$s = \frac{1}{2}(p + p')\,l + a + a'$ (LXXXVI).

$v = \frac{1}{3} h \left(\frac{a\,e - a'\,e'}{e - e'}\right)$ (LXXXVII);

$v = .2618\,h\,(d^2 + d'^2 + \sqrt{d\,d'})$ (LXXVIII).

EXAMPLES.

1. What are the surface and solid contents of the frustum of a right cone whose slant hight is 20 inches, the diameter of the ends being 4 and 2 inches respectively?

SOLUTION. Here the radii are 2 and 1, and the difference of the radii, 1 inch. Hence, hight $= \sqrt{20^2 - 1^2} = 19.9749$. By formula LXXXVIII, $v = .2618 \times h\,(d^2 + d^2 + \sqrt{dd'}) = .2618 \times 19.9749\,(4^2 + 2^2 + \sqrt{4 \times 2}) = 119.3776 +$ cu. in. By formula LXXXVI, $s = \frac{1}{2}(p + p')\,l + a + a') = \frac{1}{2} \times 18.8496 \times 20 + 12.5664 + 3.1416 = 204.204$ square inches.

NOTE. For the last operation look at formulas XXV, and XXX.

2. What are the surface and solid contents of a frustum of a regular hexagonal pyramid, the edges of its end being 6 and 4 yards respectively, and its slant hight 24 yards?

SOLUTION. By formula XXII, areas of ends are found as follows: $a = \frac{1}{2}\,a\,n\,s = \frac{1}{2} \times 5.19615 \times 6 \times 6 = 93.5307$; $a' = \frac{1}{2}\,a\,n\,s = \frac{1}{2} \times 3.4641 \times 6 \times 4 = 41.5692$.

The difference of the apothems of hexagons whose sides are 6 and 4 $= 5.1961 - 3.4641 = 1.732$; hence, by formula VII, hight of frustum $= \sqrt{24^2 - 1.732^2} = \sqrt{576 - 3\text{ (nearly)}} = 23.937$. Now, formula LXXXV, $v = \frac{1}{3}\,h\,(a + a' + \sqrt{a\,a'}) =$

$7.979\ (135.0999 + \sqrt{3887.996374}) = 1575.476$ cubic yards. And, formula LXXXVI, $s = \frac{1}{2}(p + p')\,l + a + a' = \frac{1}{2}(36 + 24)\,24 + 93.5307 + 41.5692 = 855.0999$ square yards, Ans.

EXERCISE XXIII.

1. Find the contents of a pentagonal frustum of a pyramid whose hight is 5 feet, each edge of the base 1 foot 6 inches, and each edge of the smaller end 6 inches.

2. How many cubic feet in a round piece of timber, the diameter of the greater end being 18 inches, and that of the smaller end 9 inches, and the length 14.25 feet?

3. What is the volume of a squared piece of timber, the length being 18 feet, each edge of the smaller end 12 inches, and each edge of the greater end 18 inches?

4. What is the volume of the frustum of a cone whose hight is one foot 8 inches, and the diameters of the two ends 2 feet 4 inches and 1 foot 8 inches respectively?

THE WEDGE.

96. Let l = length of back, and b = breadth of back, e = length of edge, and h = hight.

FORMULA.

$$v = \tfrac{1}{6}\, bh\, (e + 2\,l) \quad \text{LXXXIX.}$$

EXAMPLE.

1. The length and breadth of base of wedge are 70 inches and 15 inches; the edge is 110 inches in length, and the hight is 17.145 inches: required its solid contents.

SOLUTION. Here $b = 15$ inches $= 1.25$ feet; 17.145 inches $= 1.42875$ feet; $e = 110$ inches $= 9\frac{1}{6}$ feet, and $l = 70$ inches $= 5\frac{5}{6}$ feet. Now, by formula LXXXIX, $v = \frac{1}{6}\, bh\ (e + 2\,l) = \frac{1}{6} \times 1.25$ feet $\times$ 1.42875 $\times$ $(9\frac{1}{6} + 2 \times 5\frac{5}{6}) = 5.767$ cubic feet, Ans.

EXERCISE XXIV.

1. Find the solid contents of a wedge, the length and breadth of the back being 20 and 12, the edge 25, and the hight 40.

2. The hight of a wedge is 24.8 inches, the length of the edge is 110 inches, the length and breadth of back 70 inches and 30 inches respectively: required the solid contents.

3. How many cubic inches in a wedge whose altitude is 14 inches, its edge 21 inches, the length of its base 32 inches, and its breadth $4\frac{1}{2}$ inches.

SPHERE AND SPHERICAL SEGMENT.

97. Let d = diameter, v = volume, s = convex surface: in case of sperical segment, let r = radius of base, h = hight of segment, d = diameter of sphere and s = convex surface. Then we have the following:

SYNOPSIS OF FORMULAS.

For sphere, $v = .5236\ d^3$ (XC); $s = \pi\ d^2$ (XCI).
" spherical segment, $v = .5236\ h\ (3\ r^2 + h^2)$ (XCII);
" " " $v = .5236\ h^2\ (3\ d - 2\ h)$ (XCIII);
" " " $s = \pi\ d\ h$ (XCIV).

1. The diameter of a sphere is 50 inches; what are its solid contents and susface?

SOLUTION. By formula XC, $v = 5236\ d^3 = 5236 \times 50^3 = 65450$ cubic inches = 37.88 — cu. feet, Ans.

By formula XCI, $s = \pi\ d^2 = 3.1416 \times 50^2 = 7854$ cubic inches = 54.54 + square feet, Ans.

2. Find the convex surface and the solid contents of a spherical segment whose hight is 2 feet, the diameter of the sphere being 5 feet.

SOLUTION. By formula XCIII, $v = 5236\ h^2(3\ d - 2\ h) = .5236 \times 4\ (3 \times 15 + 2 \times 2) = 23.0384$ cubic feet.

By formula XCIV, $s = \pi\ d\ h = 3.1416 \times 5 \times 2 =$ 31.416 square feet.

EXERCISE XXV.

1. Find the solidity of a sphere whose diameter is 16 feet.

2. Find the surface of a globe whose diameter is 24 feet.

3. Find the volume of a sphere whose radius is 2 feet.

4. Find the solid contents or volume of the earth allowing its mean diameter to be 7912 miles.

5. What is the volume of a spherical segment the hight being 4 feet and the diameter of the base 14 feet?

6. Find the surface of a sphere whose diameter is 15 inches.

7. Find the surface of a spherical segment whose hight is 4 feet, the diameter of the sphere being 16 feet.

SPHERICAL ZONE.

98. Let r and r' = radii of the ends, d = diameter of sphere, and d' = diameter of middle zone, h = hight, and s = convex surface. Then we have the following:

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SYNOPSIS OF FORMULAS.

$v = \frac{\pi h}{2}(r^2 + r'^2 + \frac{1}{8}h^2)$ (XCV):

For middle zone,—$v = \frac{\pi h}{4}(d'^2 + \frac{2}{3}h^2)$ (XCVI);

$v = \frac{\pi h}{4}(d^2 - \frac{1}{3}h^2)$ (XCVII);

$s = \pi d h$ (XCVIII).

EXAMPLES.

1. Find the solidity of a spherical zone whose hight is 5 inches, the diameter of the sphere being 25 inches.

SOLUTION. Here by formula XXXIV, the diameter of the zone $= k = \sqrt{r^2 - a^2} = 20$ inches.

NOTE. Examine section 41 carefully and draw a figure of a circle 25 inches in diameter: now a line drawn at right angles to the diameter and 5 inches from its extremity will be the chord of an arc of a great circle, or one whose plane passes through the center. This chord will be the diameter of the zone. And $12\frac{1}{2} - 5 = 7\frac{1}{2} =$ apothem required in the formula.

Now $r = 20 \div 2 = 10$, and $r' = 0$. Then formula XCV gives $v = \frac{\pi h}{2}(r^2 + r'^2 + \frac{1}{8}h^2) = \frac{3.1416 \times 5}{2}(10^2 + 0^2 + \frac{1}{8} \times 5^2) = 850.85$ cubic inches, Ans.

2. Find the solidity of a spherical zone whose

hight is 3 feet, the diameter of the ends being 8 feet and 10 feet.

SOLUTION. By formula XCV, $v = \frac{\pi h}{2}(r^2 + r'^2 + \frac{1}{3}h^2) = \frac{3.1416 \times 3}{2}(5^2 + 4^2 + \frac{1}{3} \times 3^2) = 207.3456$ cu. feet, Ans.

EXERCISE XXVI.

1. Find the convex surface of a spherical zone whose hight is 3 feet, the diameter of the sphere being 9 feet.

2. Find the solidity of a spherical zone whose hight is 4 inches, the radii of the ends being 7 inches and 9 inches.

3. Find the solidity of a middle spherical zone whose hight is 5 feet, the diameter of the end being 8 feet.

4. Find the volume of a spherical zone whose hight is 4 feet, the diameter of its ends being 6 feet.

5. Find the volume of the middle zone of a sphere whose hight is 7 feet, the diameter of the sphere being 12 feet.

6. How many solid feet in a zone whose greater diameter is 12 feet, and less diameter 10 feet, the hight being 2 feet.

SPHEROID AND SEGMENT OF SPHEROID.

99. Let t = transverse and k = conjugate axis, h = hight of segment. Then we have the following:

SYNOPSIS OF FORMULAS.

For Oblate Spheroid,—$v = .5236\ k\ t^2$ (XCIX);
For Prolate Spheroid,—$v = .5236\ t\ k^2$ (C);
For Circular Segment of Oblate Spheroid,—

$$v = .5236\ (3k - 2h)\ \frac{t^2 h^2}{k^2} \quad \text{(CI)};$$

For Circular Segment of Prolate Spheroid,—

$$v = .5236\ (3t - 2h)\ \frac{k^2 h^2}{t^2} \quad \text{(CII)};$$

For Elliptical Segment of Oblate Spheroid,—

$$v = .5236\ (3t - 2h)\ \frac{k\ h^2}{t} \quad \text{(CIII)};$$

For Elliptical Segment of Prolate Spheroid,—

$$v = .5236\ (3k - 2h)\ \frac{t\ h^2}{k} \quad \text{(CIV)}.$$

EXERCISE XXVII.

1. What is the solidity of an oblate spheroid whose longer axis is 100 feet, and shorter axis 6 feet?

2. What is the solidity of a prolate spheroid whose axes are 40 inches and 5 inches.

3. If the axes of a prolate spheroid be 10 and 6 inches, respectively, what is the contents of

the segment whose hight is one inch, its base being parallel to the revolving axis?

4. What is the volume of an elliptical segment of a prolate spheroid, whose diameters are 11 and 15 feet; the hight of the segment being 6 feet?

5. Find the solid contents of an elliptical segment of an oblate spheroid whose diameters are 20 and 24 inches, respectively; the hight of the segment being 5 inches.

MIDDLE FRUSTUM OF SPHEROID.

100. Let l = length of frustum, and d and d' = diameters of ends. Then we have the following

SYNOPSIS OF FORMULAS.

For Circular Frustum of Oblate Spheroid,—

$$v = .2618\, l\, (2\, t^2 + d^2) \quad \text{(CV)};$$

For Circular Frustum of Prolate Spheroid,—

$$v = 2618\, l\, (2\, k^2 + d^2) \quad \text{(CVI)};$$

For Elliptical Frustum of *Oblate* or *Prolate* Spheroid,—

$$v = .2618\, l\, (2\, t\, k + d\, d') \quad \text{(CVII)}.$$

EXERCISE XXVIII.

1. Find the solid contents of the middle circular frustum of an oblate spheroid the

longer axis of the spheroid being 25 inches, the end diameter 20 inches, and the length 9 inches.

2. Find the solid contents of an elliptical middle frustum of a spheroid whose axes are 30 and 50 inches, respectively; the end diameters of the frustum being 18 and 30 inches, and its length 40 inches.

3. Find the solid contents of a circular middle frustum of a prolate spheroid whose middle or conjugate axis is 20 inches, the end diameter of the frustum being 15 inches, and its length 30 inches.

PARABOLOID; FRUSTUM OF PARABOLOID.

101. Let a = area of base of paraboloid, and h = hight; also, let a and a′ = areas of ends of frustum, and d and d′ = their diameters. Then we have the following:

SYNOPSIS OF FORMULAS.

For Paraboloid,—$v = \frac{1}{2} a h = \frac{1}{2} \times .7854 d^2 h = .3927 d^2 h$ (CVIII);

For Frustum of Paraboloid,—$v = \frac{1}{2} h (a + a')$ (CIX), or $v = .3927 h (d^2 \times d'^2)$ (CX).

EXERCISE XXIX.

1. Find the solidity of a paraboloid whose

hight is 10 inches, the diameter of the base being 20 inches.

2. Find the solid contents of a frustum of a paraboloid whose end diameters are 30 and 24 inches, the hight of the frustum being 9 inches.

HYPERBOLOID; FRUSTUM OF HYPERBOLOID.

102. Let r = radius of base and d = diameter half way between the base and vertex, and h = hight. And for frustum, let r and r′ = radii of ends, and d = diameter of section half way between ends. Then we shall have the following,

SYNOPSIS OF FORMULAS.

For Hyperboloid,—$v = .5236\,(r^2 + d^2)\,h$ (CXI);

For Frustum of Hyperboloid,—$v = .5236\,(r^2 + r'^2 + d^2)\,h$ (CXII).

EXERCISE XXX.

1. Find the solidity of a hyperboloid whose hight is 50 inches, the diameter of the base 104 inches, and the middle diameter 68 inches.

2. Find the solid contents of the frustum of a hyperboloid whose middle diameter is 40 inches, end diameters 20 and 50 inches, and hight 42 inches.

GAUGING.

Gauging is the art of measuring the capacity of vessels, such as casks, vats, etc.

Casks are usually divided into four varieties, which are easily distinguished by the curvature of their sides.

1. The middle frustum of a spheroid belongs to the first variety.

2. The middle frustum of a parabolic spindle belongs to the second variety.

3. The two equal frustums of a paraboloid belong to the third variety.

4. And the two equal frustums of a cone belong to the fourth variety.

If the contents of any of these be found in *inches*, and this be divided by 231, or 2150.4, the quotient will be the contents in *gallons* or bushels, respectively.

NOTE. Let it be observed that in all cases of gauging, where the decimal .7854 is a factor, and 231, or 2150.4 is a divisor, the decimal .0034 in the first case, and .003365 in the second, may be taken as a factor, once for all. For $.7854 \div 231 = .0034$, and $.7854 \div 2150.4 = .000365$, nearly.

Hence, for contents of cylinder in gallons, whose measurements are diameter 16 inches, and hight 3 feet, $16^2 \times 36 \times .0034 = 256 \times 36 .0034 = 31.334+$ gallons.

And if bushels were required instead of gallons, we would have $16^{c} + 36 \times .000365 = 3.36 +$ bushels.

Also, for finding the capacity of a box or bin, in bushels, multiply together the length, breadth and hight in *feet*, and the product by $\frac{4}{5}$ or .8; for $1728 \div 2150.4 = .8$, nearly.

HEAPED MEASURE.

Cob corn, potatoes, apples, etc., are sold by heaped measure. A heaped bushel contains 2509 cubic inches; a cubic foot therefore is $1728 \div 2509 = .63$, nearly, of a heaped bushel.

Hence, multiply the number of *cubic feet* by .63, for heaped bushels.

Suppose a bin, 6 feet $\times$ 8 feet $\times$ 4 feet to be level full of apples. There would be $6 \times 8 \times 4 \times .63 = 120.96$ bushels of apples.

To Estimate the Weight of Hay.

Hay is sold by the ton, but it is often inconvenient to weigh it.

1 ton	new-mown	hay	= 25	cubic	yards.
"	stored	"	= 20	"	"
"	well settled	"	= 16	"	"
"	baled	"	= 10	"	"

TO ESTIMATE THE WEIGHT OF LIVE CATTLE.

The superficial feet of animals are estimated by multiplying the feet of girth just behind the shoulder-blade, by the feet of length of back from tail to fore part of shoulder-blade.

Thus an ox whose girth is 6 feet 4 inches, and length 5 feet 9 inches, would measure $6\frac{1}{3} \times 5\frac{3}{4} = \frac{19}{3} \times \frac{23}{4} = \frac{437}{12} = 36\frac{5}{12}$ superficial feet. This is, of course, but an approximation.

Girth less than 3 feet, 11 lbs. per superficial foot.
" from 3 to 5 " 16 " " "
" " 5 to 7 " 23 " " "
" " 7 to 9 " 31 " " "

For the above instance, therefore, we would take 23 lbs. as the weight per superficial foot, and the weight of the ox would be $\frac{437}{12} \times \frac{23}{1} = 837\frac{7}{12}$ lbs.

THE CARPENTER'S RULE.

103. This instrument is used in measuring, and in artificers' work.

One face of the rule is divided into inches and eighths of inches. On the same side or face are several plane scales divided by diagonal lines into twelfths. These are chiefly used in

planning dimensions which are taken in feet and inches. The edge of the rule is divided decimally; that is, each foot is divided into ten equal parts, and each of these, again, into ten equal parts. By means of this last scale, dimensions are taken in feet, tenths and hundredths; these may be multiplied as common decimal numbers.

On some rules are found logarithmic lines for finding the contents of timber, and for gauging wine and ale. The use of these is best learned by practice.

In some rules there is, also, a slider, the back part of which will be seen to be divided like the edge of the rule, and all measuring, when it is drawn out, three feet in length.

THE BOARD RULE.

104. This rule is generally made of hickory and is about an inch wide, a quarter of an inch thick, and three feet long. On each side are three scales of numbers, showing how many feet of lumber are in boards an inch thick, of 12, 14, 16, 18, 20 and 22 feet in length, respectively.

If the boards to be measured are thinner or

thicker than one inch, parts or multiples of the rule number must be taken; or, what amounts to the same thing, the rule number must be multiplied by the thickness in inches.

For example, if the board is twelve feet long and one inch thick, and by applying the rule its width is found to be *twelve* on the scale for such board, it will contain twelve feet of lumber; but, if the board is twelve feet long and one and a half inches thick, and the measurement by the rule on the scale for a board of such length shows twelve, it will contain eighteen feet of lumber.

To find this last result, we had but to add *one-half* to the *rule number*. Had the board been two inches thick, we would have doubled the *rule number*, and we would have had twenty-four feet of lumber.

105. Let it be carefully noted that a *foot* of *lumber* consists of a board an *inch thick* and a *foot square; this is the unit of measure.* It follows that in an inch board twelve feet long, every inch in width gives a *foot of lumber*, or *board:* and, hence, the divisions on the scale for 12 feet long are an inch apart: those in the scale for 16 feet long are three-fourths of an inch apart, etc.

THE LOG RULE.

106. This rule is in material and make like the Board Rule. But the numbers on the scales represent the number of feet, *board measure*, a log of 12, 14, 16, 18, 20, or 22 feet in length contains. This rule is applied always to the smaller end of the log, and on the line of the diameter.

This and the Board Rule are often combined, one side being devoted to the measuring of boards, and the other to the measuring of logs.

A little practice enables the skillful measurer to find the number of feet in boards of any length and thickness, and in logs of any length.

TIMBER MEASURE.

107. In the absence of the Board Rule, the contents, in *board measure*, may readily be found by the rules already given in the mensuration of solids.

But let l = length in feet, w = width in *inches*, d = depth, or thickness in *inches*, and q = the number of feet of lumber. Then we have the following,—

SYNOPSIS OF FORMULAS.

For *inch* board,—$q = \frac{l\,w}{12}$ (CXIII).

For *any thickness*,—$q = \frac{l\,w\,d}{12}$ (CXIV).

EXERCISE XXXI.

1. How many feet of lumber in a board 12 feet long, an inch thick and 13 inches wide?

2. How many feet of lumber in ten pieces of flooring 12 feet long, $1\frac{1}{2}$ inches thick, and whose several widths are 6, 5, 8, 6, $7\frac{1}{2}$, 8, 5, $4\frac{1}{2}$, 7 and 5 inches?

3. How many feet of lumber in nine two-inch planks 16 feet long, whose several widths are 13, 17, 14, 11, 10, 14, $12\frac{1}{2}$, 16 and $9\frac{1}{2}$ inches?

4. How many feet of lumber in a plank two inches thick, whose breadth at one end is 15 inches and at the other 17 inches, the length 6 feet?

5. How many feet of lumber in fifteen hundred 3x4 scantlings 18 feet long?

6. How many feet of lumber in a load of half-inch siding 14 feet long, whose aggregate width is 2016 inches?

7. How many feet of lumber in 40 rafters three inches thick and twenty-one feet long,

whose narrow ends are four, and wide ends seven, inches?

NOTE 1. The mean width is found by taking half the sum of the two end widths. Where boards are sawn of unequal end widths it is customary to take the middle measurement for its mean width.

NOTE 2. Lumber is sold at so much per 1000 feet. Hence, for finding the value of lumber, cut off three places from the right, or if there are decimals, remove the decimal point three places to the left; this will reduce the number of feet to thousands and decimals of a thousand. Multiply this by the price per *M*.

Of course, if there be not three places to cut off, prefix as many ciphers as will make it read thousands; then multiply by the price per *M*.

NOTE 3. The lumberman soon becomes accustomed to the following process:—If the price is $10 a thousand, he cuts off two figures from the right, or removes the decimal point two places to the left; this gives the price of the lumber at once. If it is $12 per thousand, he adds a fifth of the result at $10. If it is $15 per thousand, he adds a half of the result at $10, etc.

8. What is the value of 23,468 feet of lumber at $10 per thousand?

9. What is the value of 23,468 feet of lumber at $12 per *M*?

10. What is the value of 23,468 feet of lumber at $20 per thousand?

11. If a board is an inch thick and 6 inches broad, what length will make a square foot, or a foot of lumber?

12. If an inch board is 16 inches broad, what length of it will make 7 feet of lumber?

ARTIFICER'S WORK.

108. Stone Masonry is measured by the cubic foot, cubic yard, or by the perch. A yard of stone wall is *three feet long, three feet wide,* and *fifteen inches thick.* A perch is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot deep.

109. Bricklaying is estimated by the thousand bricks, or by the rod of $272\frac{1}{4}$ square feet, the standard thickness being a brick and a half. If a wall be of any thickness, it must be reduced to this standard; this is done by multiplying the area of the wall by the number of half bricks in the thickness, and dividing the product by three.

110. Paving is measured by the square foot, or the square yard.

111. Flooring, partitioning, shingling, tiling and slating are measured by the square of 100 feet. 1000 shingles are estimated to the square.

112. Glazing is measured by the square foot, including the sash. Circular windows are meas-

ured as if they were square, taking for their dimensions their greatest length and breadth.

113. Plastering is measured by the square foot or square yard, including doors and windows.

Whitening and *coloring* are measured in the same manner.

Papering is measured by the piece.

114. Painting is measured by the square yard. Every part is measured where the color lies, and the measuring line is forced into all the mouldings and corners. Double measure is allowed for curved mouldings.

Windows are done for so much a piece.

Sash frames for so much a dozen.

Sky-lights, *window-bars*, casements, etc., are charged at a certain price per piece.

CONSPECTUS OF FORMULAS.

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1	$A = \frac{b\,p}{2}$	12	Bk. IV. Prop. 6.	
2	$b = \frac{2\,A}{p}$	12	Derived from No. 1.	
3	$p = \frac{2\,A}{b}$	12	Derived from No. 1.	
4	$A = \sqrt{m\,(m-a)\,(m-b)\,(m-c)}$	12	Mensuration, Case III.	
5	$A = \frac{b^2}{4}\sqrt{3} = .433\,b^2$	12	Mensuration, Case III.	139
6	$h = \sqrt{b^2 + p^2}$	12	Bk. IV., Prop. II.	
7	$b = \sqrt{h^2 - p^2}$	12	Derived from No. 6.	
8	$p = \sqrt{h^2 - b^2}$	12	Derived from No. 6.	
9	$s' = \frac{p^2}{h}$	12	Bk. IV., Prop. 23.	
10	$s = \frac{b^2}{h}$	12	Bk. IV., Prop. 23.	
11	$p' = \sqrt{s\,s'}$	12	Bk. IV., Prop. 23, Cor.	

12	$A = b^2$	21	Bk. IV, Prop. 5.	
13	$b = \sqrt{A}$	21	Derived from No. 12.	
14	$A = \frac{d^2}{2}$	21	Bk. IV, Prop. 11. Cor. 4.	
15	$d = \sqrt{2A}$	21	Derived from No. 14.	
16	$A = b\,p$	21	Bk. IV, Prop. 5.	
17	$b = \frac{A}{p}$	21	Derived from No. 16.	
18	$p = \frac{A}{b}$	21	Derived from No. 16.	
19	$A = b\ \sqrt{(d+b)(d-b)}$	22		189
20	$A = \frac{p}{2}\sqrt{b+b'}$	22	Bk. IV, Prop. 7.	
21	$A = \frac{d}{2}\sqrt{1+l'}$	22		189
22	$A = \frac{a\,n\,s}{2}$	27	Bk. V, Prop. 8.	
23	$s = \frac{2A}{a\,n}$	27	Derived from No. 22.	
24	$n = \frac{2A}{a\,s}$	27	Derived from No. 22.	
25	$c = \pi\,d$	34	Bk. V, Prop. 16.	

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26	$d = \frac{c}{\pi} = .3183\ c$	34	Derived from No. 25.	
27	$A = \frac{c\,d}{4}$	34	Bk. V, Prop. 15.	
28	$d = \frac{4\,A}{c}$	34	Derived from No. 27.	
29	$c = \frac{4\,A}{d}$	34	Derived from No. 27.	
30	$A = \pi\,r^2$	34	Bk. V, Prop. 16.	
31	$r = \sqrt{\frac{A}{\pi}} = \sqrt{A \times .3183}$	34	Derived from No. 30.	
32	$A = d^2 \times .7854$	35		139
33	$A = .0796\ c^2$	35		139
34	$d^2 = .3183^2 \times c^2$	35		139
35	$A = \frac{\pi}{4}(d + d')(d - d')$	37		139
36	$A = .0796\ (c + c')(c - c')$	37		140

37	$A = \frac{1}{4}(c + c')(d - d')$	37		140
38	$l = \frac{\pi n d}{360} = .608726\ n\ d$	38		140
39	$k = 2\sqrt{r^2 - a^2}$	39		140
40	$a = \frac{1}{2}\sqrt{4r^2 - k^2}$	39		140
41	$k' = \sqrt{2r(r - a)}$	39		140
42	$r = \frac{k'^2}{2h}$	39		140
43	$A = \frac{l r}{2}$	41	Bk. V, P. 12.	
44	$A = .008726\ n\ r$	42	Derived from Nos. 38 and 43.	
45	Area $= A \pm A'$	42	Mens. of Surfaces § 107.	
46	Area $= \frac{2}{3} k h + \frac{h^2}{2k}$	42		141
47	Area $= A - A'$	42		
48	$x = a \times .8862269$	48		141
49	$x = a \times .2820948$	49		141
50	$x = a \times .7071068$	49		141
51	$x = \sqrt{a \times .6366197}$	50		141
52	$x = a \times 1.4142136$	50		142

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53	$x = a \times 4.4428934$	50		142
54	$x = a \times 1.1283791$	51		142
55	$x = a \times 3.5449076$	51		142
56	$c = \pi\sqrt{\frac{t^2 + k^2}{2}}$	55		142
57	$t = \frac{k\,a}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k^2}{4} - o^2}\right\}$	55		142
58	$k = \frac{o\,t}{\sqrt{a\,(t-a)}}$	56		143
59	$o = \frac{k}{2}\sqrt{a\,(t-a)}$	56		143
60	$a = \frac{t}{2} \pm \frac{t}{k}\sqrt{\frac{k^2}{4} - o^2}$	56		143
61	$A = \frac{\pi\,t\,k}{4} = .7854\,t\,k$	56		143
62	$A = \frac{2\,a\,b}{3}$	62		144

63	$l = 2\sqrt{o^2 + \frac{4}{3}a^2}$	62	146
64	$p = \frac{o^2}{a}$	62	146
65	$A = \frac{4ka}{75t}\left\{3\sqrt{7a(7t \times 5a)} + 4\sqrt{ta}\right\}$	66	146
66	$t = \frac{ka}{o^2}\left\{\frac{k}{2} \pm \sqrt{\frac{k^2}{4} + o^2}\right.$	67	146
67	$k = \frac{ot}{\sqrt{a(t+a)}}$	67	147
68	$a = d \pm \frac{t}{2}$	67	147
69	$o = \frac{k}{t}\sqrt{a(t+a)}$	67	147
70	$s = e^2\sqrt{3} = 1.732\ e^2$	75	147
71	$v = \frac{1}{2}e^3\sqrt{2} = .11785\ e^3$	75	147
72	$s = 6\ e^2$	75	147
73	$v = e^3$	75	147
74	$s = 2\ e^2\sqrt{3} = 3.464\ e^2$	75	148
75	$v = \frac{1}{3}e^3\sqrt{2} = .471405\ e^3$	75	148
76	$s = 15\ e^2\sqrt{\frac{1}{8}(5 + 2\sqrt{5})} = 20.645775\ e^2$	75	148

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77	$v = 5\ e^3 \sqrt{\frac{47+21\sqrt{5}}{40}} = 7.6631\ e^3$	75		149
78	$s = 5\ e^2\ \sqrt{3} = 8.66\ e^2$	75		151
79	$v = \frac{5}{6}\ e^3\ \sqrt{\frac{1}{2}(7+3\sqrt{5})} = 2.18169\ e^3$	75		151
80	$v = h\ a$	78	Bk. VII, Prop. 14.	
81	$s = h\ p + 2\ a$	78	Bk. VII, Prop. 14.	
82	$s = h'\ p + 2\ a$	78		152
83	$v = \frac{1}{3}\ h\ a$	81	Bk. VIII, Prop. 5.	
84	$s = \frac{1}{2}\ p\ l + a$	81	Bk. VIII, Prop. 3.	
85	$v = \frac{1}{3}\ h\ (a + a' + \sqrt{a\ a'})$	83	Bk. VII, Prop. 18, Cor.	
86	$s = \frac{1}{2}\ l\ (p + p') + a + a'$	83	Bk. VII, Prop. 4.	
87	$v = \frac{1}{3}\ h\ \left(\frac{a\ e - a'\ e'}{e - e'}\right)$	83		152
88	$v = .2618\ h\ (d^2 + d'^2 + \sqrt{d\ d'})$	83	Bk. VIII, Prop. 6.	
89	$v = \frac{1}{6}\ b\ h\ (e + 2\ l)$	86	Mens. Solids, § 121.	

90	$v = .5236\ d^3$	87	Bk. VIII, Prop. 14, C. 2.	
91	$s = \pi\ d^2$	87	Bk. VIII, Prop. 10, C. 1.	
92	$v = .5236\ h\ (3\ r^2 + h^2)$	87		153
93	$v = .5236\ h^2\ (3\ d - 2\ h)$	87		153
94	$s = \pi\ d\ h$	87	Bk. VIII, Prop. 10, C. 2.	
95	$v = \frac{\pi\ h}{2}\ (r^2 + r'^2 + \frac{1}{3}\ h^2)$	89		153
96	$v = \frac{\pi\ h}{4}\ (d'^2 + \frac{2}{3}\ h^2)$	89		153
97	$v = \frac{\pi\ h}{4}\ (d^2 - \frac{1}{3}\ h^2)$	89		154
98	$s = \pi\ d\ h$	89	Bk. VIII, Prop. 10, C. 2.	
99	$v = .5236\ k\ t^2$	91	Davies' Dict. Math.	
100	$v = .5236\ k^2\ t$	91	Davies' Dict. Math.	
101	$v = .5236\ (3\ k - 2\ h)\ \frac{t^2\ h^2}{k^2}$	91		154
102	$v = .5236\ (3\ t - 2\ h)\ \frac{k^2\ h^2}{t^2}$	91		154
103	$v = .5236\ (3\ t - 2\ h)\ \frac{k\ h^2}{t}$	91		154
104	$v = .5236\ (3\ k - 2\ h)\ \frac{t\ h^2}{k}$	91		154

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106	$v = .2618\ l\ (2\ k^2 + d^2)$	92		155
107	$v = .2618\ l\ (2\ t\ k + d\ d')$	92		155
108	$v = \frac{1}{2}\ a\ h = .3927\ d^2\ h$	93		155
109	$v = \frac{1}{2}\ h\ (a + a')$	93		155
110	$v = .3927\ h\ (d^2 + d'^2)$	93		155
111	$v = .5236\ (r^2 + d^2)\ h$	94		155
112	$v = .5236\ (r^2 + r'^2 + d^2)\ h$	94		155
113	$q = \frac{l\ w}{12}$	101		
114	$q = \frac{l\ w\ d}{12}$	101		

MNEMONICS.

I have found it convenient often, to give pupils an opportunity to apply a system of mnemonics to certain larger numbers, such as those involved in the formulas for equivalent areas; XLVIII, XLIX, etc.

Let only consonants have a value, as follows:

t = 1	c	b	f
n = 2	k	v	p
m = 3	g	w	h
r = 4	q	j	
	= 7	= 8	= 9
l = 5			s
d = 6			z
			= 0

When the consonants have been substituted for the numbers, put vowels in anywhere, so as to make up a word. The vowels will have no value, and the word will almost always be meaningless. For example, take formula XLVIII, and the seven immediately following:

$x = a \times .\text{bewodenénadof} = a \times .8862269.$
$x = a \times .\text{nevenezéferow} = a \times .2820948.$
$x = a \times .\text{cazacatézadow} = a \times .7071068.$
$x = \sqrt{a \times .\text{demedadétapec}} = \sqrt{a \times .6366197.}$
$x = a \times 1.\text{riterinítamid} = a \times 1.4142136.$
$x = a \times 4.\text{rarinavápimer} = a \times 4.442884.$
$x = a \times 1.\text{tinivimicápit} = a \times 1.1283791.$
$x = a \times 8.\text{liriripézicad} = a \times 8.5449076.$

In memorizing these we say for the first of the above, — x = a multiplied by decimal bewodenénadof, etc., etc.

It is surprising how easily one can remember such words.

Take formula XXXII,—A = d^2 × .caviler, *i. e.*, Area = d^2 multiplied by decimal cáviler = d^2 × .7854.

Take again, formula XXXIII,—A = .sighed × c^2, *i. e.*, A = decimal sighed multiplied by c^2.

In the case of 7, 8, 9, and 0, several letters have the same power, hence a choice is afforded, and when one letter will not give a euphonious word, another can be substituted for it. Still, I think sometimes, the more jaw-breaking the word the more easily it is remembered.

TABLE III.

OF THE AREAS OF THE SEGMENTS OF A CIRCLE, WHOSE DIAMETER IS 1, AND SUPPOSED TO BE DIVIDED INTO 1000 EQUAL PARTS.

Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.
.001	.000042	.030	.006865	.059	.018766	.088	.033872
.002	.000119	.031	.007209	.060	.019239	.089	.034441
.003	.000219	.032	.007558	.061	.019716	.090	.035011
.004	.000337	.033	.007913	.062	.020196	.091	.035585
.005	.000470	.034	.008273	.063	.020681	.092	.036162
.006	.000618	.035	.008438	.064	.021168	.093	.036741
.007	.000779	.036	.009008	.065	.021659	.094	.037323
.008	.000951	.037	.009383	.066	.022154	.095	.037909
.009	.001135	.038	.009763	.067	.022652	.096	.038496
.010	.001329	.039	.010148	.068	.023154	.097	.039087
.011	.001533	.040	.010537	.069	.023659	.098	.039680
.012	.001746	.041	.010931	.070	.024168	.099	.040276
.013	.001968	.042	.011330	.071	.024680	.100	.040875
.014	.002199	.043	.011734	.072	.025195	.101	.041476
.015	.002438	.044	.012142	.073	.025714	.102	.042080
.016	.002685	.045	.012554	.074	.026236	.103	.042687
.017	.002940	.046	.012971	.075	.026761	.104	.043296
.018	.003202	.047	.013392	.076	.027289	.105	.043908
.019	.003471	.048	.013818	.077	.027821	.106	.044522
.020	.003748	.049	.014247	.078	.028356	.107	.045139
.021	.004031	.050	.014681	.079	.028894	.108	.045759
.022	.004322	.051	.015119	.080	.029435	.109	.046381
.023	.004618	.052	.015561	.081	.039979	.110	.047005
.024	.004921	.053	.016007	.082	.030526	.111	.047632
.025	.005230	.054	.016457	.083	.031076	.112	.048262
.026	.005546	.055	.016911	.084	.031629	.113	.048894
.027	.005867	.056	.017369	.085	.032180	.114	.049528
.028	.006194	.057	.017831	.086	.032745	.115	.050165
.029	.006527	.058	.018296	.087	.033307	.116	.050804

Table III.

Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.
.117	.051446	.156	.078194	.195	.107842	.234	.139841
.118	.052090	.157	.078921	.196	.108636	.235	.140688
.119	.052736	.158	.079649	.197	.109430	.236	.141537
.120	.053385	.159	.080380	.198	.110226	.237	.142387
.121	.054036	.160	.081112	.199	.111024	.238	.143238
.122	.054689	.161	.081846	.200	.111823	.239	.144091
.123	.055345	.162	.082582	.201	.112624	.240	.144944
.124	.056003	.163	.083320	.202	.113426	.241	.145799
.125	.056663	.164	.084059	.203	.114230	.242	.146655
.126	.057326	.165	.084801	.204	.115035	.243	.147512
.127	.057991	.166	.085544	.205	.115842	.244	.148371
.128	.058658	.167	.086289	.206	.116650	.245	.149230
.129	.059327	.168	.087036	.207	.117460	.246	.150091
.130	.059999	.169	.087785	.208	.118271	.247	.150953
.131	.060672	.170	.088535	.209	.119083	.248	.151816
.132	.061348	.171	.089287	.210	.119897	.249	.152680
.133	.062026	.172	.090041	.211	.120712	.250	.153546
.134	.062707	.173	.090797	.212	.121528	.251	.154412
.135	.063389	.174	.091554	.213	.122347	.252	.155280
.136	.064074	.175	.092313	.214	.123167	.253	.156149
.137	.064760	.176	.093074	.215	.123988	.254	.157019
.138	.065149	.177	.093836	.216	.124810	.255	.157890
.139	.066140	.178	.094601	.217	.125634	.256	.158762
.140	.066833	.179	.095366	.218	.126459	.257	.159636
.141	.067528	.180	.096134	.219	.127285	.258	.160510
.142	.068225	.181	.096903	.220	.128113	.259	.161386
.143	.068924	.182	.097674	.221	.128942	.260	.162263
.144	.069625	.183	.098447	.222	.129773	.261	.163140
.145	.070328	.184	.099221	.223	.130605	.262	.164019
.146	.071033	.185	.099997	.224	.131438	.263	.164899
.147	.071741	.186	.100774	.225	.132272	.264	.165780
.148	.072450	.187	.101553	.226	.133108	.265	.166663
.149	.073161	.188	.102334	.227	.133945	.266	.167546
.150	.073874	.189	.103116	.228	.134784	.267	.168430
.151	.074589	.190	.103900	.229	.135624	.268	.169315
.152	.075306	.191	.104685	.230	.136465	.269	.170202
.153	.076026	.192	.105472	.231	.137307	.270	.171089
.154	.076747	.193	.106261	.232	.138150	.271	.171978
.155	.077469	.194	.107050	.233	.138995	.272	.172867

Table III.

Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.
.273	.173758	.312	.209227	.351	.245934	.390	.283592
.274	.174649	.313	.210154	.352	.246889	.391	.284568
.275	.175542	.314	.211082	.353	.247845	.392	.285544
.276	.176435	.315	.212011	.354	.248801	.393	.286521
.277	.177330	.316	.212940	.355	.249757	.394	.287498
.278	.178225	.317	.213871	.356	.250715	.395	.288476
.279	.179122	.318	.214802	.357	.251673	.396	.289453
.280	.180019	.319	.215733	.358	.252631	.397	.290432
.281	.180918	.320	.216666	.359	.253590	.398	.291411
.282	.181817	.321	.217599	.360	.254550	.399	.292390
.283	.182718	.322	.218533	.361	.255510	.400	.293369
.284	.183619	.323	.219468	.362	.256471	.401	.294349
.285	.184521	.324	.220404	.363	.257433	.402	.295330
.286	.185425	.325	.221340	.364	.258395	.403	.296311
.287	.186329	.326	.222277	.365	.259357	.404	.297292
.288	.187234	.327	.223215	.366	.260320	.405	.298273
.289	.188140	.328	.224154	.367	.261284	.406	.299255
.290	.189047	.329	.225093	.368	.262248	.407	.300238
.291	.189955	.330	.226033	.369	.263213	.408	.301220
.292	.190864	.331	.226974	.370	.264178	.409	.302203
.293	.191775	.332	.227915	.371	.265144	.410	.303187
.294	.192684	.333	.228858	.372	.266111	.411	.304171
.295	.193596	.334	.229801	.373	.267078	.412	.305155
.296	.194509	.335	.230745	.374	.268045	.413	.306140
.297	.195422	.336	.231689	.375	.269013	.414	.307125
.298	.196337	.337	.232634	.376	.279982	.415	.308110
.299	.197252	.338	.233580	.377	.270951	.416	.309095
.300	.198168	.339	.234526	.378	.271920	.417	.310081
.301	.199085	.340	.235473	.379	.272890	.418	.311068
.302	.200003	.341	.236421	.380	.273861	.419	.312054
.303	.200922	.342	.237369	.381	.274832	.420	.313041
.304	.201841	.343	.238318	.382	.275803	.421	.314029
.305	.202761	.344	.239268	.383	.276775	.422	.315016
.306	.203683	.345	.240218	.384	.277748	.423	.316004
.307	.204605	.346	.241169	.385	.278721	.424	.316992
.308	.205527	.347	.242121	.386	.279694	.425	.317981
.309	.206451	.348	.243074	.387	.280668	.426	.318970
.310	.207376	.349	.244026	.388	.281662	.427	.319959
.311	.208301	.350	.244980	.389	.282617	.428	.320948

Table III.

Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.	Hight.	Area Seg.
.429	.321938	.447	.339798	.465	.357727	.483	.375702
.430	.322928	.448	.340793	.466	.358725	.484	.376702
.431	.323918	.449	.341787	.467	.359723	.485	.377701
.432	.324909	.450	.342782	.468	.360721	.486	.378701
.433	.325900	.451	.343777	.469	.361719	.487	.379700
.434	.326892	.452	.344772	.470	.362717	.488	.380700
.435	.327882	.453	.345768	.471	.363715	.489	.381699
.436	.328874	.454	.346763	.472	.364713	.490	.382699
.437	.329866	.455	.347759	.473	.365712	.491	.383699
.438	.330858	.456	.348755	.474	.366710	.492	.384699
.439	.331850	.457	.349752	.475	.367709	.493	.385999
.440	.332843	.458	.350748	.476	.368708	.494	.386699
.441	.333836	.459	.351745	.477	.369707	.495	.387699
.442	.334829	.460	.352742	.478	.370706	.496	.388699
.443	.335822	.461	.353739	.479	.371705	.497	.389699
.444	.336816	.462	.354736	.480	.372764	.498	.390699
.445	.337810	.463	.355732	.481	.373703	.499	.391699
.446	.338804	.464	.356730	.482	.374702	.500	.392699

Table IV.

A gallon of water weighs 10 lbs. avoirdupois.
A cubic foot of water weighs 62½ lbs.=1000 ozs.
A pail of water=2½ gallons=25 lbs.
A gallon is equal to 231 cubic inches.
A gallon dry measure contains 268.8 cubic inches.
A bushel contains 2150.42 cubic inches.

Table V.

7.92 inches=1 link.
100 links=1 chain.
80 chains=1 mile.
10,000 sq. links=1 square chain.
10 sq. chains=1 acre.
1 chain=4 rods=66 feet.
1 acre=160 sq. rods=4840 sq. yards.

MISCELLANEOUS PROBLEMS.

1. There is a cistern seven feet deep whose top and bottom are equal and circular; the diameter being 8 ft., what is its capacity in barrels?

2. A certain lodging chamber was 16 feet long, 14 feet wide, and 6 feet 10 inches in hight at the center,—but following the roof the ceiling met the floor at the sides. Now, it is stated in our text-books on Hygiene, that a man requires at least seven cubic feet of pure air per minute. How long would the air in this room supply four men?

3. A school-room is 30 feet square and 9 feet high. Allowing seven cubic feet of pure air to each pupil per minute, how long will this room supply sixty children?

NOTE.—No wonder the children in many school-rooms are sleepy and stupid, and in bad temper.

4. A room is 18 feet long, and 14 feet wide; how many yards of carpet $\frac{5}{8}$ of a yard wide would be required to cover it?

5. How many feet of lumber in a tight fence surrounding a garden eight rods square, the

fence being 6 feet high, and the boards $\frac{3}{4}$ of an inch thick?

6. How many cords of wood in a pile 96 feet long, 34 feet wide, and 10 feet high?

7. The diagonal of a field is 20 rods; what is its area?

8. An embankment 12 feet high and $4\frac{1}{2}$ feet thick, contains 6,000 cubic feet; what is its length?

9. A room in the form of a cube contains 15,625 cubic feet; how far is it from one corner on the floor to the corner diagonally opposite on the ceiling?

10. A room is 25 feet long, 20 feet wide, and 12 feet high; how far from one lower corner to the opposite upper corner?

11. A ladder 40 feet long will reach to a window 32 feet high on one side of a street, and on the other side, another window 24 feet high; what is the breadth of the street?

12. A room is 20 feet long, 18 feet wide, and 12 feet high; how many yards of carpeting $\frac{3}{4}$ of a yard wide would it take to carpet it; and, how many yards of paper $\frac{5}{8}$ of a yard wide would be required to paper it, deducting 30 square feet for doors and windows?

13. How many cubic feet of air in a room $14\frac{1}{2}$ feet long, $12\frac{1}{2}$ feet wide, and 10 feet high?

How many yards of carpet would be required to cover the floor, the carpet being a yard wide?

14. A lecture room is 47 feet long, 37 feet wide, and 9 feet high. It is seated for 300 persons. Now, allowing 7 cubic feet of pure air to each person per minute, how long will there be a supply for such an audience?

NOTE.—People are sometimes required to sit an hour and a half, and even two hours in such a place! What filth is there taken into the lungs! Enter such a place when lecture is half over, and get a taste of what they are breathing!

15. The edge of a cube is 18 inches; what is the edge of one containing 8 times as much?

16. There is in a certain tannery a circular vat, 11 feet in diameter at the bottom, and 10 feet in diameter at the top. There is also a vat of the form of a parallelopiped 8 feet long, 5 feet wide, and 4 feet high. How many times the capacity of the latter is the former?

17. What must be the dimensions, in feet, of a bin whose width and hight are to be equal, and half its length, to contain 1,000 bushels of corn?

18. How many cubic inches of iron would be required to make 1,000 12-inch solid shot?

19. In a given month 9 inches of rain fell; what must have been the depth of a cistern whose diameter was 10 feet, to hold the water

which fell on the roof of a house 36 feet long, and 24 feet wide, the ridge being 9 feet above the beam, the eaves and ends of the roof extending 18 inches beyond the building, horizontal measurement? It is granted that the cistern did not leak, the water did not evaporate, and that none was taken out during the time.

NOTE.—Only so much rain can fall on a certain area, whether it be received on a flat surface, or an inclined surface, within the same limits. Hence, in this case the incline of the roof must be disregarded.

20. A man's farm is in the form of a square, containing 250 acres; what is the distance in rods from one corner to the opposte corner?

21. A man bought a village lot containing half an acre running from the middle of the street. His frontage was 72 feet; how far back from his street fence did the lot extend, it being in the form of a rectangle, and the street being 50 feet wide?

22. How many yards of carpet would be required for a square room whose diagonal is 31 feet?

23. The area of a reservoir in a rectangular shape is 36,320 square yards; one side is 94 yards, what is the other?

24. Find the area of a parallelogram whose base is 120 rods, and altitude 45 rods.

25. What is the length of the base of a triangle whose area is 2,560 square yards and altitude 40 yards?

26. Required the hight of a triangle which contains 117.5625 square yards, its base being 49½ feet in length.

27. Find the area of a triangular field whose sides are 12, 18, 24 rods, respectively.

28. Find the area of an equilateral flower-bed whose sides are each 15 feet long.

29. The four sides of a quadrilateral inscribed in a circle are 45, 30, 50, and 65 chains, respectively; what is its area?

NOTE.—From half the sum of the four sides subtract each side separately; the square root of the product of the four remainders will give the area.—See demonstration at the end of the supplement.

30. There is a quadrilateral whose opposite angles together are equal to two right angles; the sides are as follow: 25, 34, 35, and 16 inches; what is its area?

NOTE.—If in a quadrilateral the sum of two angles diagonally opposite each other be equal to two right angles, the figure may be circumscribed by a circle.

31. The area of a park in the form of an octagon is 69 acres, 2 roods, 4.6 rods; the apothem is 57.94 rods, what is the length of a side?

32. What is the circumference of a circle whose diameter is 15 yards?

33. What is the diameter of a circle whose circumference is 32 986$\frac{8}{}$ inches?

34. Find the area of a circular park whose diameter is 200 yards, in terms of acres and the lower denominations.

35. Find the area of a circular garden whose circumference is 200 feet.

36. What is the area of a sector of a circle whose radius is 50 yards, the arc of the sector being 30 yards in length?

37. The circumference of a circular field is 400 perches; what is the side of a square field of equal area?

38. Find the area of a rectangular field whose length and breadth are respectively 740 yards and 180 yards.

39. A circular garden is to be formed so as to contain as much land as an equilateral triangle whose side is 56 chains. Find diameter and area of the garden.

40. The auditorium of a church is 48 feet long, 30 feet wide, and 16 feet high. Now, allowing seven cubic feet of pure air per minute to each person, how long will there be a supply of fresh air for an audience of 250 persons?

41. There are two columns in the ruins of Persepolis left standing upright: the one is 64

feet above the plain, and the other 50: in a straight line between these stands an ancient small statue, the head of which is 97 feet from the summit of the higher, and 86 feet from the top of the lower column, the base of which measures just 76 feet from the centre of the figure's base. Find the distance between the tops of the two columns.

42. The perambulator, or surveying wheel, is so constructed as to turn just twice in the length of a pole, or $16\frac{1}{2}$ feet; find the diameter.

43. In turning a sulky within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one: the wheels were both $4\frac{1}{2}$ feet high. Now supposing them to be 5 feet apart on the axle-tree, what was the circumference of the track described by the outer wheel?

NOTE.—Circumferences of circles are to each other as their radii, Use an algebraic symbol for one of the radii, and make a proportion.

44. A roof, which is 28 feet 6 inches by 15 feet 4 inches, is to be covered with lead, at 8 pounds per square foot; what will it cost at $3.60 per cwt.?

45. If a round pillar, 7 inches in diameter, have 4 feet of stone in it, of what diameter is

the column of equal length that contains ten times as many cubic feet?

46. A circular fish pond is to be made in a garden that shall inclose half an acre; what must be the length of the cord that strikes the circle?

47. The distance between the centres of two circles, whose diameters are each 50 inches, being given equal to 30 inches, what is the area of the space enclosed by the parts of their circumferences that interlap?

48. The area of an equilateral triangle, whose base falls on the diameter, and its vertex in the middle of the arc of a semi-circle, is equal to 100 square inches; what is the diameter of the circle?

49. What is the length of a chord which cuts off one fourth of the area of a circle whose diameter is 78 inches?

50. How many 4 inch cubes may be cut out of a 12 inch cube?

51. What would it cost to paint a conical spire at $15 per square yard, supposing the hight to be 115 feet and the diameter of the base 18 feet?

52. The standard unit of dry measure is the bushel, which contains 2,150.4 cubic inches, or 77.6274 pounds of distilled water at its maximum density. Now the bushel measure is a

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cylinder 18½ inches in diameter; what is its depth?

53. What would it cost to gild a ball for a flag pole, at 1½ cents per square inch, the diameter of the ball being 5 inches.

54. What will the frustum of a marble cone cost at $2.40 per solid foot, the diameter of the larger end being 3 feet, and of the smaller, 1½ feet, and the slant hight 6 feet?

55. A cone is to be divided into three equal parts by planes parallel to the base; find the altitude of each part, the entire cone being 20 inches high.

NOTE.—Remember that *all similar* solids are to each other as the *cubes* of their like dimensions. Proceed, therefore, by means of proportion.

56. If an ox, whose girth is 6 feet, weighs 600 pounds, what is the weight of an ox whose girth is 8 feet?

57. Four women own a ball of butter, 5 inches in diameter. It is agreed that each shall take her share separately from the surface of the ball; how many inches of the diameter shall each one take?

58. A cubic foot of brass is to be drawn into wire of $\frac{1}{10}$ an inch in diameter; what will the length of the wire be, allowing no loss in the metal in the working.

59. The ditch of a fortification is 1,000 feet long, 9 feet deep, 20 feet broad at bottom, and 22 feet broad at the top; how many barrels of water would be required to fill it?

60. How many cubic feet in a stick of timber whose length is 24½ feet, and the mean breadth and thickness each 1.04 feet?

61. Find the length of a circular arc of 68°, the diameter of the circle being 250 feet.

62. Find the area of a sector of a circle whose diameter is 100 feet, the arc of the sector containing 70°.

63. Find the area of a regular decagon whose side is 11 inches, and apothem 9.526279 inches.

64. What is the length of an arc of 197½° of a circle, whose diameter is 240 feet?

65. What is the circumference of an ellipse whose diameters are 36 and 24 feet?

66. A field containing 7 acres 3 roods 21 perches 17 yards is divided into two parts, the one forming a circle whose diameter is 80 yards; what must be the dimensions of an equilateral triangle whose area shall be equal to the remainder?

67. Find the area of a circular annulus contained between two circles whose circumferences are 360 and 240 chains.

68. What is the area of an ellipse whose diameters are 5 and 10 feet, respectively?

69. The axes of an ellipse are 30 inches and 10 inches, and one abscissa is 24 inches; what is the ordinate?

70. The conjugate axis of an ellipse is 10 feet, the smaller abscissa is 6 feet, and the ordinate 4 feet; what is the transverse axis?

71. The transverse axis of an ellipse is 280 inches, an ordinate 80 inches, and an abscissa 56 inches; what is the conjugate axis?

72. The ordinate of a parabola is 20 inches, and the corresponding abscissa 36 inches; what is the parameter?

73. Find the area of a parabola whose base is 15 inches, and hight, or abscissa, 22 inches.

74. Find the length of a parabolic curve whose abscissa is 6 inches, and ordinate 12 inches.

75. The transverse axis of an hyperbola is 15 inches, the conjugate axis 9 inches, the smaller abscissa 5 inches; what is the ordinate?

76. Find the area of an hyperbola whose axes are 15 inches and 9 inches, and the smaller abscissa 52 inches.

77. Find the volume and surface of a tetrahedron whose edge is 8 inches.

78. What is the surface of a right cylinder

whose length is 20 inches, and circumference 6 inches?

79. What are the volume and surface of a regular pentagonal pyramid, each side of its base being 2 feet, and its slant hight 10 feet?

80. Find the surface of a sphere whose diameter is 8 feet.

81. What is the volume of a prism whose length is 18 feet, its base being a regular hexagon whose side is 16 inches, and apothem 13.8564 inches?

82. Find the surface of a spherical segment whose hight is 2 inches, the diameter of the sphere being 10 inches.

83. If the volume of a triangular prism is 7.6545 cubic inches, and its hight $10\frac{1}{2}$ inches, what is the area of its base?

84. Required the volume of the frustum of a square pyramid, the side of the greater base being 16 inches, and of the lesser base 10 inches, the hight being 18 inches.

85. What is the solidity of a sphere whose diameter is 30 inches?

86. What is the diameter of the sphere whose volume is 65449.95 feet?

87. Find the volume of a middle zone of a sphere, the hight being 12 inches, and end diameters 8 inches.

88. Find the solidity of an oblate spheroid whose axes are 20 inches and 12 inches, respectively.

89. How many pails of water may be contained in a circular cistern whose diameter is 7 feet, and depth 11 feet?

90. What must be the depth of a pentagonal cistern which contains as much water as a circular cistern 8 feet deep, and a rectangular tank 7 feet long, 5 feet wide, and $3\frac{1}{2}$ feet deep,—one side of the pentagonal cistern being 5 feet?

91. Find the diameter of a circle which shall contain as much ground as a quadrilateral inscribed in a circle, whose four sides are 900, 1000, 600, and 800 yards, respectively.

92. What is the length of each side of a square field which contains five acres?

93. How many yards of matting would cover a floor, the dimensions of which are 20 feet 10 inches by 15 feet $5\frac{1}{2}$ inches?

94. How much will it cost to carpet a parlor 18 feet square, with carpeting $\frac{3}{4}$ of a yard wide, at $1.50 per yard?

95. How many bushels will a box 8 feet long, 4 feet wide, and 3 feet high contain?

96. The standard unit of liquid measure is the gallon (wine measure), containing the number

of cubic inches represented by its prime factors, 3, 7, and 11. What is that number, and what would be the dimensions of a vessel that would contain one gallon?

97. What is the length in feet and inches of each side of a square carpet, made from $49\frac{1}{2}$ yards of Brussels carpeting $\frac{3}{4}$ yard wide?

98. How long is a field containing 14 acres if it is 35 rods wide?

99. How many square rods are there in a lot $15\frac{1}{2}$ rods long and $12\frac{3}{7}$ rods wide?

100. What is the length of a side of a cubical box which contains 389017 solid inches?

101. How many yards of cloth $\frac{1}{2}$ of a yard wide are equivalent to 12 yards $\frac{3}{4}$ of a yard wide?

102. An open court contains 40 square yards; how many stones 9 inches square will be required to pave it?

103. How many sq. yds. of paving are there in a street 2700 ft. long and 40 ft. wide?

104. What is the cost of a field 77 rods long and 41 rds. wide at $17.60 an acre?

105. A man had a yard 38 ft. long and 27 ft. wide; he reserved two grass plats each 8 ft. square, and had the rest paved with stone at 45 cents a square yard; what did the paving cost?

106. What would be the cost of enough oil-

cloth to cover a room 12x16½ feet at 75 cents per square yard?

107. At 35 cents per square yard, what would it cost to plaster a wall 15 feet high and 54 feet long?

108. How many linear yards of carpeting 1½ yards wide will cover a floor 18 feet square?

109. When snow is uniformly 6 inches deep, how many cubic feet are there on one acre of land?

110. If 32⅔ sq. yds. of carpeting will cover a floor 14 ft. wide, what is the length of the floor?

111. If a load of wood is 8 ft. long and 3 ft. wide, how high must it be to contain a cord?

112. How many cu. ft. in a rectangular beam 24 ft. 6 in. long, 1 ft. 9 in. wide, and 1 ft. 2½ inches thick?

113. How many cords of wood in a pile 140 ft. long, 4½ ft. wide and 6½ ft. high?

114. Required the area in acres, etc., of a piece of land .5 of a mile long and .3 of a mile broad.

115. How much will it cost to dig a cellar 40 ft. long, 32 ft. wide, and 5 ft. deep, at 25 cents a cubic yard?

116. The four sides of my garden are 168 feet, 280 feet, 182 feet, and 252 feet, respec-

tively. Suppose that each board is 8 inches wide, and that the fence is 5 boards high; how many feet of inch lumber will be required to fence it?

117. What is the length, in feet and inches, of each side of a square carpet, made from 208⅓ yards of Brussels carpeting ¾ yard wide?

118. What is the length of the side of a cubical block which contains 389017 solid inches?

119. If 20 yds. of cloth ¾ yd. wide are required for a dress, what must be the width of a piece 12 yds. long to answer the same purpose?

120. How many acres are there in a street 4 rods wide and 2½ miles long?

121. How many gallons of water will a cistern hold which is 7 feet long, 6 feet wide, and 11 feet deep?

122. How many bricks will it require to build a wall 2 rods long, 6 feet high, and 18 inches thick, each brick being 8 inches long, 4 inches wide, and 2½ inches thick?

123. What will it cost to lay a pavement 36 ft. long, and 9 ft. 6 in. wide, at 40 cents a square yard?

124. What will it cost to dig a cellar 40 ft. long, 21 ft. 6 in. wide, and 4 ft. deep, at $1.75 a cubic yard?

125. A flag-pole 180 feet high casts a shadow 135 feet in length; what is the distance from the top of the pole to the end of the shadow?

126. The four walls of a room are each 16 feet in length and 9 feet in hight, and the ceiling is 16 feet square; how much will it cost to plaster it, at 14 cents a square yard?

127. How many feet of inch lumber is there in a board 17 ft. 6 in. in length, and 1 ft. 7 in. in width?

128. How many cords are there in three piles of four-foot wood, the first 36 feet long and 4 feet high, the second 42 feet long and 5 feet high, the third 20 feet long and 6 feet high?

129. What would it cost to enclose a square lot containing 160 acres, with a fence costing at the rate of $4 per rod?

130. How much carpeting $\frac{3}{4}$ of a yard wide is required for a room 27 ft. 3 in. long, and 22 ft. 6 in. wide?

131. If 251 A. 65 P. of land are laid out in the form of a square, what will be the length of each side?

132. At 36 cents per square yard for plastering, and 75 cents per roll for paper hanging, how much will it cost to plaster and paper the walls of a room 18x16x9 feet, making allowance,

in papering, for two windows, each 3x6 feet, and three doors, each 3x7 feet, the paper being 1 foot 6 inches wide, and 7 yards in a roll?

133. How many cords of wood in a pile 196 feet long, 7 feet 6 inches high, and 8 feet wide?

134. A pile of wood is 256 feet long, 8 feet high, and 16 feet wide; what would be the side of a cubical pile containing the same quantity?

135. A certain room is 27 feet long, 18 feet wide, and 10 feet high. How many pieces of paper $\frac{1}{2}$ yard wide (9 yards in a piece) will the side walls require, no allowance being made for doors, windows, etc.? Also, how many yards of carpeting $\frac{3}{4}$ of a yard wide would be needed for the same room?

136. In Troy, Pa., there is a cistern seven feet deep whose top and bottom are equal, and in the form of a sector of a circle, the length of the arc being 15 feet, and the radius 8 feet; what is its capacity in barrels?

SUPPLEMENT.

V. In an equilateral triangle A B C, let C be the vertex and A B the base. Draw C D perpendicular to the base A B. It will bisect it.

Now, $\overline{B\,C}^2 - \overline{D\,B}^2 = \overline{C\,D}^2$. But $\overline{D\,B}^2 = \frac{\overline{B\,C}^2}{4}$; since the square of half a line is always equal to a fourth of the square of a whole line. Hence

$\left(\overline{B\,C}^2 - \frac{\overline{B\,C}^2}{4}\right)\frac{\overline{B\,C}^2}{4} = \overline{C\,D}^2 \times \frac{\overline{B\,C}^2}{4}$, the square of the area. And, the square root of the first number of this equation, $\sqrt{\left(\overline{B\,C}^2 - \frac{\overline{B\,C}^2}{4}\right)\frac{\overline{B\,C}^2}{4}} = \sqrt{\frac{3\,\overline{B\,C}^4}{16}} = \frac{\overline{B\,C}^2}{4}\sqrt{3} = .433\ b^2$.

XIX. $\sqrt{(d+b)\,(d-b} = \sqrt{d^2 - b^2} = p$; hence the formula.

XXI. The formula is evident from a consideration of the areas of the two triangles into which the diagonal divides the figure.

XXXII. The areas of circles are to each other as the squares of their diameters. Hence,—$2^2 : d^2 :: 3.1416 : A = \frac{d^2 \times 3.1416}{4} = .7854\ d^2$

XXXIII. $d = \frac{c}{\pi}$ (xxv) $= .3183\ c$; therefore $A = (.3183\ c)^2 \times .7854 \times .0796\ c^2$.

XXXIV. $d = .3183\ c$; hence, $d^2 = (.3183\ c)^2$.

XXXV. Let d be the larger circle, and d′, the diame-

ter of the smaller circle. Now $d^2 \times .7854$ = area of larger ole, and $d'^2 \times .9854$ = area of smaller ole. The difference of these two expressions gives us the formula.

XXXVI. Here c and c′ representing the circumferences of the two oles, we have from XXXIII, their respective areas, $.0796\ c^2$, and $.0796\ c^2$. The difference of these gives the formula.

XXXVI. Apply XXVII, as above, and find $A = \frac{1}{4}(c\,d - c'\,d') = \frac{1}{4}(c + c')(d - d')$. For multiply $(c + c')$ by $(d - d')$, and we have $c\,d + c'.d - c\,d' - c'\,d'$. But, in comparing the parts of the two oles, we have $c : c' :: d : d'$, hence $c'\,d = c\,d'$. And canceling these out of the product, we have the formula.

XXXVIII. Here $\pi\,d$ would equal circumference of the ole; $\pi\,d \div 360$ = a degree of it, $\pi\,n\,d \div 360$ = length of any arc = $.008726\ n\ d$.

XXXIX. Let AB be a chord in a circle, ABM. From the centre draw CD perpendicular to AB, and terminating in the middle of the arc ADB, at D, and join CB and CA. Now CA, CB, and CD are radii, and each equal to r. Designate the point of intersection of CD with AB by E. CE = a, the apothem. $EB = \sqrt{r^2 - a^2}$, and AB, which is double of EB, equals $2\sqrt{r^2 - a^2}$.

XL. Assuming the figure indicated above, we have $a^2 = r^2 - \left(\frac{k}{2}\right)^2 = r - \frac{k^2}{4} = \frac{1}{4}(4\,r^2 - k^2)$, and $a = \frac{1}{2}\sqrt{4\,r^2 - k^2}$.

XLI. From the same figure, $\overline{DB}^2 = \overline{EB}^2 + \overline{DE}^2 = (r^2 - a^2) + (r - a)^2$; hence $\overline{DB} = k' = \sqrt{2\,r\,(r - a)}$.

XLII. In the above figure, $k'^2 = h^2 + r^2 - (r - h)^2$; whence $r = \frac{k'^2}{2h}$.

XLVI. This formula was discovered by Dr. Chas. Hutton, by means of fluxions. Its demonstration would be too tedious here.

XLVIII. In XXXII, we have $A = d^2 \times .7854$. Let d become unity and $A = .7854$. The side of a square equal in area to a circle having unity for diameter $= \sqrt{.7854} = .8862269$. Now $1^2 : d^2 :: .7854 : d^2 \times .7854$. But $.7854 : d^2\ .7854 :: \sqrt{.7854} : d\ \sqrt{.7864}$. Whence the Rule.

XLIX. In XXXIII, $A = .0796\ c^2$. Let c become unity: then $A = .0796$ or, more exactly, $A = .079577$. The side of a square equal in area to this would be $\sqrt{.079577} = .2820908$.

Now as the areas of circles are as the squares of their like elements, $-1^2 : c^2 :: .079577 : c^2 \times .079577$; and $.079577 : c^2 \times .079577 :: \sqrt{.079577} : c\ \sqrt{.079577}$. Whence $c \times .2820948 = a \times .2820848$.

L. If the diameter is unity, the area of the circumscribed square is one, and the inscribed square is one-half (Legendre, Bk. 5, Prop. 12). The side of the inscribed square is, then, $\sqrt{\frac{1}{2}} = .7071068$. This will increase directly as the diameter increases. Hence the Formula.

LI. Let the area of the circle be unity. Now we have $1 : d :: \pi : d\pi = c$. Then $d = \frac{4A}{c}$ (XXXIII) $= \frac{4A}{d\pi}$. And $d^2 = \frac{4A}{\pi}$: Therefore $d = 2\sqrt{\frac{A}{\pi}}$. And $\frac{d}{2} = \sqrt{\frac{A}{\pi}}$. But as the area of the circle is to be unity in the discussion, we shall have $\frac{d}{2} = \sqrt{\frac{1}{\pi}} = r$; and $r^2 = \frac{1}{\pi}$. But, $2r^2 = \frac{2}{\pi} = x^2 = \frac{2}{3.1416} = .6366197$ = the area of the

inscribed square. Now $1 : a :: .6366197 :: x^2 \therefore x^2 = a \times .6366197$, and $x = \sqrt{a \times .6366197}$.

LII. Legendre, Bk. V, Prop. 3. The radius is to the side of the inscribed square as 1 is to $\frac{1}{2}\sqrt{2}$. Hence, when the side of the square is unity, the radius of the circumscribed circle is .7071068, and the diameter is $.7071068 \times 2 = 1.4142136$. Whence the Formula.

LIII. From reference to Legendre given in LII, it will appear that when the side of the inscribed square is unity, the diameter of the circumscribed circle is 1.4142136; and, hence, its circumference is $1.4142136 \times 3.1416 = 4.4428934$. Hence the Formula.

LIV. In demonstrating (LI), it was shown that $d = 2\sqrt{\frac{A}{\pi}}$. But assuming the side of the inscribed square to be unity, its area will be unity. Hence, $d = 2\sqrt{\frac{1}{\pi}} = \sqrt{\frac{5}{3.1416}} = 1.1283791$. Hence the formula.

LV. Here we have only to multiply 1.1283891 by 3.1416, which gives us 3.5449076. Hence the formula.

LVI. The geometrical mean between the two axes of an ellipse is equal to the diameter of a circle equal in area to the ellipse; but the arithmetical mean exceeds the geometrical mean, while the circumference of the ellipse exceeds that of the circle equal in area to it; therefore $\frac{t+k}{2} \times \pi = (t+k) \times \frac{1}{2}\pi$ will give the circumference, nearly. But the arithmetical mean; viz: $\pi\sqrt{\frac{t^2+k^2}{2}}$ as in the formula is more exact.

LVII. It is a property of the ellipse, that—$t^2 : k^2 \therefore (a\,t - a^2) : o^2\ o^2\ t^2 = k^2\ a\,t - k^2\ a^2$; by transposition,

$o^2 t^2 - k^2 a t = - k^2 a^2$, and by dividing both sides of the equation by o^2, we have $t^2 - \frac{k^2 a}{o^2} t = - \frac{k^2 a^2}{o^2}$; completing the squares we have, $t^2 - \frac{k^2 a}{o^2} x + \frac{k^4 a^2}{40^4} = \frac{k^4 a^2}{40^4} - \frac{k^2 a^2}{o^2}$; $t = \frac{k^2 a}{20^2} \pm \sqrt{\frac{k^4 a^2}{40^4} - \frac{k^2 a^2}{o^2}}$; hence, $t = \frac{k a}{o^2} \times \left(\frac{k}{2} \pm \sqrt{\frac{k^2}{4} - o^2}\right)$, the form of the given formula.

LVIII. From the equation of the ellipse, we have $t^2 : k^2 :: a (t - a) : o^2$; then $t : k :: \sqrt{a (t - a)} : o$, whence $k = \frac{o t}{\sqrt{a (t - a)}}$.

LIX. From the above proportion (LVIII),—$t^2 : k^2 :: a (t - a) : o^2$, we have by taking the square root of all the terms, $t : k :: \sqrt{a (t - a)} : o$, and hence, $o = \frac{k}{t} \sqrt{a (t - a)}$.

LX. As above, $t^2 : k^2 :: a (t - a) : o^2$; $o^2 t^2 = k^2 a t - k^2 a^2$; dividing by k^2, we have $a t - a^2 = \frac{t^2 o^2}{k^2}$; hence, $a^2 - t a = - \frac{t^2 o^2}{k^2}$; completing the squares and extracting the square root of both numbers, we have, on reducing, $a = \frac{t}{2} \pm \frac{t}{k} \sqrt{\frac{k^2}{4} - o^2}$.

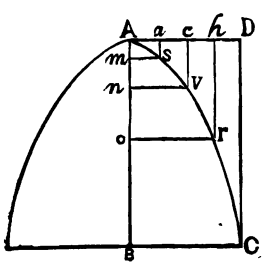
LXI. The square of half the transverse axis is to the square of half the conjugate, as the rectangle of any two abscissas to the square of the ordinate which divides them (Derived from equation (e) article 105, Church's Analytical Geometry); or Figure XL. $\overline{A O}^2 : \overline{C O}^2 :: A f' \times f' B : \overline{L' f'}^2$. Now, if upon either axis a circle

be described, the corresponding circular and elliptic ordinates will be proportional; because the rectangles of the abscissas are equal to the squares of the circular ordinates. And the sum of all the circular ordinates, or the area of the circle, is to the sum of all the elliptic ordinates, or the area of the ellipse, as the diameter upon which the circle is described, to the conjugate diameter.

Therefore, the ellipsis is a mean proportional between the circumscribed and inscribed circles. For, let C = area of circumscribing circle and E = area of ellipse, c = area of inscribed circle, and let A B, and C D be the transverse and conjugate axes, respectively. Then we shall have,—C : E :: A B : C D; and E : c :: A B : C D. Therefore C : E :: E : c.

And, again, a circle whose diameter is a mean proportional between the axes, is equal to the ellipse. For, from the last proportion we have $E^2 = C\,c$; hence, $E = \sqrt{C\,c}$. But $C = \frac{\pi\,t^2}{4}$, and $c = \frac{\pi\,k^2}{4}$, hence $E = \sqrt{\frac{\pi^2\,t^2\,k^2}{16}} = \frac{\pi\,t\,k}{4} = 7854\,t\,k$.

LXII. The area of a parabola may be shown to be equal to ⅔ of its circumscribing parallelogram in the following manner:



In the figure draw C D parallel to A B, and conceive A D to be divided into an indefinite number of equal parts in the points a, c, h, etc., through which conceive a series of parallels to be drawn, such as a s, c v, h r, etc., meeting the semi-ordinates m s, n v, o r, etc., in the curve, at the points s, v, r, etc.

Then from the property of the curve, viz: that the squares of the ordinates of any two points in a parabolic curve are proportional to the corresponding abscissas (Church's Anal. Geometry, art. 89). we have,—

$$A\,B : \overline{B\,C}^2 :: A\,m : \overline{m\,s}^2$$
$$A\,B : \overline{B\,C}^2 :: A\,n : \overline{n\,v}^2$$
$$A\,B : \overline{B\,C}^2 :: A\,o : \overline{o\,r}^2, \text{ etc.}$$

But $A\,m = a\,s$, $A\,n = c\,v$, $A\,o = h\,r$, $A\,B = D\,C$; therefore, by inversion, we have,—

$$\overline{B\,C}^2 : D\,C :: \overline{o\,r}^2 : h\,r$$
$$\overline{B\,C}^2 : D\,C :: \overline{n\,v}^2 : c\,v$$
$$\overline{B\,C}^2 : D\,C :: \overline{m\,s}^2 : a\,s, \text{ etc.}$$

In these proportions, $\overline{m\,s}^2$, $\overline{n\,v}^2$, $\overline{o\,r}^2$, etc., are a series of squares, whose roots A a, A c, A h, etc., are in arithmetical progression, beginning with 0 at A, the common difference being 1, and number of terms infinite; and as the lines a s, c v, h r, etc., are as the squares, having C D the greatest term, and A D, the number of terms.

Now, in a series of squares whose sides or roots form an arithmetical progression, differing by 1, and commencing with a cipher, the sum, when the series is infinite, is equal to $\frac{1}{3}$ of the greatest term multiplied by the number of terms. Thus,—

1. $0 + 1 + 4 = 5$. But $4 \times 3 = 12$; then $\frac{5}{12} = \frac{1}{3} + \frac{1}{12}$.
2. $0 + 1 + 4 + 9 = 14$. But $9 \times 4 = 36$; then $\frac{14}{36} = \frac{1}{3} + \frac{1}{18}$.
3. $0 + 1 + 4 + 9 + 16 = 30$. But $16 \times 5 = 80$; then $\frac{30}{80} = \frac{1}{3} + \frac{1}{24}$.

From which it will be seen that the fraction in excess of $\frac{1}{3}$ diminishes as the number of terms increases. therefore, the number of terms is infinite, the fr less than any assignable quantity, and the rule stated will hold true. Hence, to proceed, we

the sum of all the lines, (C D × A D) ÷ 3; but A B × B C = C D × A D; therefore (A B × B C) ÷ 3 = the sum of all the lines which constitute the space A D C A, outside of the semi-parabola. But the area of the parallelogram A B C D is A B × B C; therefore A B × B C — (A B × B C) ÷ 3 = ⅔ (A B × B C) = the area of the semi-parabola B A r C. Hence the area of the whole parabola is ⅔ A B × E C, *i. e.*, two-thirds of the circumscribing parallelogram.

LXIII. This rule is an approximation derived by a process involving differential calculus. It is not thought prudent to introduce the demonstration here.

LXIV. Every ordinate of a parabola is a mean proportional between its abscissa and the *latus rectum*, or parameter (Church's Analytical Geometry, art. 84); hence, $a : o :: o : p$; Then, $p = \frac{o^2}{a}$.

LXV. This rule is an approximation drawn from the equation of the hyperbola. Its presentation here would be tedious and of little use.

LXVI. In the hyperbola it may be shown that the square of half the transverse is to the square of half the conjugate axis, as the rectangle of the abscissa, and the sum of the abscissa and the transverse axis, to the square of the ordinate; or,—

$\left(\frac{t}{2}\right)^2 : \left(\frac{k}{2}\right)^2 :: a\,(t+a) : o^2$; hence, $t^2 : k^2 :: a\,t + a^2 : o^2$; Also, $o^2\,t^2 = k^2\,a\,t + k^2\,a^2$, and, $o^2\,t^2 - k^2\,a\,t = k^2\,a^2$; dividing by o^2 we have $t^2 - \frac{k^2\,a}{o^2}\,t = \frac{k^2\,a^2}{o^2}$. Now, completing the square and extracting the square root of both members of this equation and reducing, we have the given Formula.

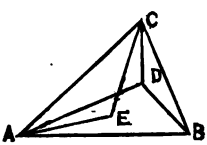
LXVII. By the last demonstration we found $t^2 : k^2 :: (t + a)\ a : o^2$. Hence, $t : k :: \sqrt{a\,(t+a)} : o$; whence, $k = \frac{o\,t}{\sqrt{a\,(t+a)}}$.

LXVIII. $d = \frac{t}{k}\sqrt{\frac{k^2}{4} + o^2}$, which is the equation of the eccentricity of the hyperbola, or the distance from the middle point of the transverse axis to either focus.

LXIX. $t^2 : k^2 :: a\,(t+a) : o^2$, hence $t : k :: \sqrt{a(t+a)} : o$, and, $o = \frac{k}{t}\sqrt{a\,(t+a)}$.

LXX. The area of an equilateral triangle is (V) $\frac{b^2}{4}\sqrt{3}$; then the area of the four sides will be $b^2\sqrt{3} = e^2\sqrt{3}$.

LXXI. Let A B C D be a tetrahedron. From C draw the perpendicular C E, to the opposite side A B D, and join E A. Then $\overline{AC}^2 = \overline{AE}^2 + \overline{EC}^2$; but $\frac{1}{3}\overline{AC}^2 = \frac{1}{3}\overline{AB}^2 = \overline{AE}^2$, since E is at the centre of the equilateral triangle A B D. Therefore, $\frac{2}{3}\overline{AC}^2 = \overline{EC}^2$. Hence $AC\sqrt{\frac{2}{3}} = EC$; but $ABD = \frac{1}{4}\overline{AB}^2\sqrt{3} = \frac{1}{4}\overline{AC}^2\sqrt{3}$; then, the solidity will be equal to the area of the base multiplied by $\frac{1}{3}$ of the altitude (Legendre, Bk. VII, Prop. XVI, Cor. 2); or, $\frac{1}{3}\,CE \times ABD = \frac{1}{3}AC\sqrt{\frac{2}{3}} \times \frac{1}{4}\overline{AC}^2\sqrt{3} = \frac{1}{12}\overline{AC}^3\sqrt{2} = \frac{1}{12}e^3\sqrt{2} = .11785\ e^3$.

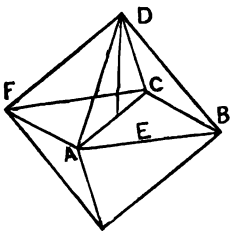


LXXII. The surface consists of six equal squares. The area of one of these squares is e^2; hence $6\ e^2$ for the entire surface.

LXXIII. The figure is a cube; hence for the volume, e^3.

LXXIV. The area of one of the faces is $\frac{e^2}{4}\sqrt{3}$, since each face is an equilateral triangle. The eight faces will then be $8 \times \frac{e^2}{4}\sqrt{3} = 2e^2\sqrt{3}$.

LXXV. In this figure, let E be the centre of the solid, or the middle of the diagonal AC; join DE, which is equal to AE. The solid is composed of two equal square pyramids having a common base ABCF, equal to the square of any edge of the solid, the altitude of each being DE, or AE, half the diagonal of that square. Now, $\overline{AB}^2 = ABCF$; but the area $ABCF \times \frac{1}{3}AE = \overline{AB}^2 \times \frac{1}{3}AC = \frac{1}{3}\overline{AB}^2\sqrt{\overline{AB}^2 + \overline{BC}^2} = \frac{1}{3}\overline{AB}^2\sqrt{2\overline{AB}^2} = \frac{1}{3}\overline{AB}^2\sqrt{2} = \frac{1}{3}e^2\sqrt{2}$.

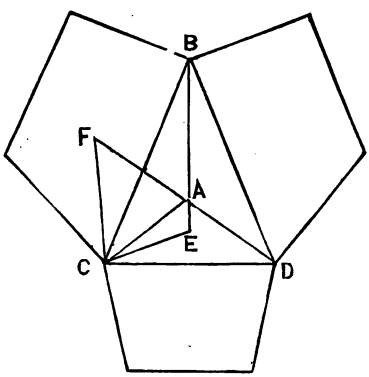


LXXVI. In table II, the area of a pentagon whose side is unity, is 1.7204774. This is equal to $\frac{5}{4}\tan. 54° = \frac{5}{4}\sqrt{1 + \frac{2}{5}\sqrt{5}}$, which being multiplied by 12 will give the whole surface, and may easily be put in the form given.

NOTE. The expression, $\frac{5}{4}\tan. 54° = \frac{5}{4}\sqrt{1 + \frac{2}{5}\sqrt{5}}$ is read five-fourths of the tangent of 54 degrees. It is found as follows: $\tan. 54° = \frac{\text{sin. } 54°}{\text{co. sin. } 54°} = \frac{\frac{1+\sqrt{5}}{4}}{\frac{\sqrt{10-2\sqrt{5}}}{4}} = \frac{1+\sqrt{5}}{4} \times \frac{4}{\sqrt{10-2\sqrt{5}}} = \frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}$; Now, squaring

both members we have $\tan.^2 54° = \frac{3 + \sqrt{5}}{5 - \sqrt{5}}$; and multiplying both terms of the fraction by the denominator taken with the contrary sign and reducing, we have $1 + \frac{2}{5}\sqrt{5}$; then extracting the square root of both members, we have, $\tan. 54° = \sqrt{1 + \frac{2}{5}\sqrt{5}}$; this multiplied by $\frac{5}{4}$ gives the expression. It is $\frac{5}{4}$ of the tangent, because the figure is a pentagon.

LXXVII. Let C B D–A be a solid angle of the dodecahedron represented, in part, in this figure, and connect the extremities of sides A B, A C, A D, of the faces which form the angle, by the lines B C, C D and D B, forming an equilateral triangle B C D, within the solid, on the centre of which let fall the perpendicular A E: join the centre F, of one of the faces, and the points A and C.



The angle C A D contains 108 degrees, being one of the angles of a regular pentagon. Its sine is $\frac{1}{4}\sqrt{10+2\sqrt{5}}$, to the radius 1.

The angle A D C contains 36 degrees, being one-third of the angle of a regular pentagon. Its sine is $\frac{1}{4}\sqrt{10-2\sqrt{5}}$.

$$EA = \sqrt{\overline{AC}^2 - \overline{CE}^2} = \sqrt{\left\{\overline{AC}^2 - \left(\frac{1+\sqrt{5}}{2\sqrt{3}}\right)^2 AC^2\right\}}$$

$$= AC\sqrt{\left(1 - \frac{3+\sqrt{5}}{6}\right)} = AC\sqrt{\frac{3-\sqrt{5}}{6}}.$$

The chord of an arc being a mean proportional between its versed sine and the diameter; AE being the versed sine whose chord is A C, and the diameter being equal to that of the circumscribing sphere, we have,—

$$\overline{AC}^2 + 2AE = \overline{AC}^2 + 2AC\sqrt{\frac{3-\sqrt{5}}{6}} = \tfrac{1}{2} AC \times$$

$$\sqrt{\frac{6}{3-\sqrt{5}}} = \tfrac{1}{2} AC\sqrt{6 \times \frac{3+\sqrt{5}}{9-5}} = \tfrac{1}{2} AC\sqrt{\left(3 \times \frac{6+2\sqrt{5}}{4}\right)} = AC \times \frac{1+\sqrt{5}}{4} \times \sqrt{3} = \frac{\sqrt{3}+\sqrt{15}}{4}, \quad AC =$$

R, the radius of the circumscribed sphere.

Again, the angle AFC contains 54 degrees, whose sine is $\frac{1+\sqrt{5}}{4}$. Hence $\sqrt{10+2\sqrt{5}} : 1+\sqrt{5} :: AC : AF =$

$$\frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}} \times AC = AC\sqrt{\frac{5+\sqrt{5}}{10}}.$$

Now it is obvious that the radius of the circumscribed sphere is the hypothenuse of a right angled triangle whose sides containing the right angle are AF and the radius of the inscribed sphere; hence we have,—

$$\sqrt{(R^2 - \overline{AF}^2)} = \sqrt{\left[\left(\frac{\sqrt{3}+\sqrt{15}}{4} \times AC\right)^2 - \frac{5+\sqrt{5}}{10} \times AC^2\right]} = AC\sqrt{\frac{25+11\sqrt{5}}{40}} = \text{radius of inscribed}$$

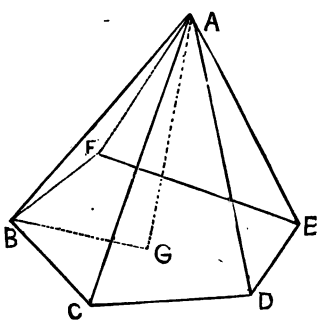
sphere. Again, the solidity of any solid is equal to the surface multiplied by $\frac{1}{3}$ of the radius of the inscribed sphere; and the surface equals $15\,e^2\sqrt{\frac{5+2\sqrt{5}}{5}}$ (LXXVI);

therefore $s \times \frac{1}{3} r = 15 e^2 \sqrt{\frac{5+2\sqrt{5}}{5}} \times \frac{1}{3} e \sqrt{\frac{25+11\sqrt{5}}{40}}$

$= 5 e^3 \sqrt{\frac{47+21\sqrt{5}}{40}}$.

LXXVIII. By formula (V), the area of one of the faces is $\frac{e^2}{4}\sqrt{3}$, it being an equilateral triangle. But the figure has twenty such faces; hence $s = \frac{20 \times e^2}{4}\sqrt{3} = 5 e^2 \sqrt{3}$.

LXXIX. Let A be a solid angle of an icosahedron.



This angle is formed by five triangles, whose bases form the pentagon B C D E F. On the centre of this pentagon let fall the perpendicular A G and join B G. In one of the steps of demonstration (LXXVII,) it was shown that $BG = AB \times \sqrt{\frac{5+\sqrt{5}}{10}}$, and the radius of the circle circumscribing one of the faces A B C, of the solid equals $A\sqrt{\frac{1}{3}}$, where A = the area of the equilateral triangle B A C. But the radius of the circumscribing sphere is $R = \frac{\overline{AB}^2}{2AG} = \frac{\overline{AB}^2}{2\sqrt{(\overline{AB}^2 - \overline{BG}^2)}} =$

$$\frac{\overline{AB}^2}{2\sqrt{\overline{AB}^2 - \frac{5+\sqrt{5}}{10}\overline{AB}^2}} = \frac{AB}{2\sqrt{(1-\frac{5+\sqrt{5}}{10})}} = 2\sqrt{\frac{5-\sqrt{5}}{10}}$$

$$\times \sqrt{\tfrac{1}{3}\overline{AB}} \times \sqrt{\frac{10}{5-\sqrt{5}}} = AB\sqrt{\left(\frac{10}{5-\sqrt{5}} \times \frac{5+\sqrt{5}}{5+\sqrt{5}}\right)}$$

$= \frac{1}{2} A B \sqrt{\frac{10 \times (5 + \sqrt{5})}{25 - 5}} = A B \sqrt{\frac{5 + \sqrt{5}}{8}}$.

Now, R is the hypothenuse of a right-angled triangle, of which one leg is Q ($= A B \sqrt{3}$), the radius of the circle circumscribing one of the faces A B C, and the other, the radius r, of the inscribed sphere. Hence $r = \sqrt{R^2 - Q^2} = \sqrt{\left(\frac{5+\sqrt{5}}{8} \times \overline{AB}^2 - \frac{1}{3} \overline{AB}^2\right)} = A B \times \sqrt{\frac{15 + 3\sqrt{5} - 8}{8 \times 3}} = A B \sqrt{\frac{7 + 3\sqrt{5}}{24}}$.

If the whole surface be denoted by s, and the solidity by v, we shall have $v = \frac{1}{3} r s = \frac{1}{3} A B \sqrt{\frac{7 + 3\sqrt{5}}{24}} \times 5 \times A B \sqrt{3} = \frac{1}{8} A B^3 \sqrt{\frac{7 + 3\sqrt{5}}{8}} = \frac{5}{8} \overline{AB}^3 \sqrt{\frac{7 + 3\sqrt{5}}{2}} = \frac{5}{8} e^3 \sqrt{\frac{1}{2}(7 + 3\sqrt{5})}$.

LXXXII. If paper were fitted snugly to the convex surface of an oblique prism and then spread out, it would be of the form of an oblique angled parallelogram; this is measured by multiplying the base by the altitude. (Legendre, Bk. IV, P. 5.)

LXXXVII. Assuming the symbols used in the text, section 95, and letting c = hight of apex, we have, for volume of frustum, $v = \frac{1}{3}(h + c) a - \frac{1}{3} c a'$. Here c forms no part of the elements of the frustum. To eliminate it we take the proportion,—

$a : a' :: (h + c)^2 : c^2$; or,

$e^2 : e'^2 :: (h + c)^2 : c^2$; hence

$e : e' :: (h + c) : c$, and by division,

$e - e' : e' :: (h + c) - c : c$; whence $c = \frac{e' h}{e - e'}$; substituting this for c in the above equation, we have, $v = \frac{1}{3} \times h \left(\frac{a e - a' e'}{e - e'}\right)$.

XCII. Assuming the symbols of section 97, we have $(d - h)\,h = r^2$ (Legendre, Bk. IV, Prop. 28, Cor.); hence $d = \frac{r^2 + h^2}{h}$. Substitute this value for d in formula XCIII, and the rule becomes as given.

XCIII. If the solidity of the cone be taken from the solidity of the sector, we shall have the solidity of the spherical segment. That is, $.5236\,d^2\,h - (d^2\,h - 3\,d\,h^2 + 2\,h^3) \times .5236 = .5236\,h^2\,(3\,d - 2\,h)$.

XCV. Subtract the lesser segment from the greater, and the difference will be the spherical zone: that is, putting m and n = hight of lesser and greater segments respectively, $.5236\,n\,(3\,r^2 + n^2) - .5236\,m\,(3\,r'^2 + m^2) = .5236\,[(3\,r^2\,n + n^3) - (3\,r'^2\,m + m^3)]$. Now, $(d - n)\,n = r^2$, and $(d - m)\,m = r'^2$. Hence, $\frac{r^2 + n^2}{n} = \frac{r'2 + m^2}{m}$. Hence, the above expression, may become $\frac{\pi\,h}{2}\left(r^2 + r'^2 + \frac{h^2}{3}\right)$.

XCVI. If in the above one of the radii pass through the centre, we get $r^2 = \frac{d^2}{4} = r'^2 + f$, where f = half the hight of the middle zone. Then the expression $\frac{\pi\,h}{2} \times \left(r^2 + r'^2 + \frac{h^2}{3}\right)$ becomes $f\,\pi\,(r'^2 + \frac{2}{3}\,f^2) = \left(\frac{d'^2}{4} + \frac{2}{3}\,f^2\right) \times f\,\pi = (d'^2 + \frac{8}{3}\,f^2)\,\frac{f\,\pi}{4}$. This is half the spherical zone. Placing h for f, we have, $\frac{h\,\pi}{4}\,(d'^2 + \frac{2}{3}\,h^2)$. For $\frac{1}{4}$ of $\frac{8}{3}\,f^2 = \frac{2}{3}\,h^2$. A square of half a line is equal to a fourth of the square of the whole line.

XCVII. If in the equation, $\frac{\pi h}{2}\left(r^2 + r'^2 + \frac{h^2}{3}\right)$, we write f for h and substitute for the quantity $(r'^2 + \frac{1}{3} f^2)$, its value, $r^2 - \frac{1}{3} f^2$, we shall have $(2 r^2 - \frac{1}{3} f^2) \times f \times 1.5708 = (r^2 - \frac{1}{3} f^2) f \pi = \left(\frac{d^2}{4} - \frac{1}{3} f^2\right) f \pi = \frac{f \pi}{4}(d^2 - \frac{4}{3} f^2)$. This measures half the middle spherical zone. But placing h instead of f, we have $\frac{\pi h}{4}(d^2 - \frac{1}{3} h^2)$; because the square of half a line is equal to a fourth of the square of the whole line.

CI–CII. The demonstration of these two formulas cannot be conveniently given here. It would lead into the subject of infinities and would be too tedious.

CIII. The area of any circular segment is to the area of the corresponding elliptical segment, as the diameter upon which the circle is described to its conjugate diameter ("Properties of the Conic Sections").

Hence the sum of all the circles which form the segment of the sphere is to the sum of all the ellipses which form the corresponding segment of the spheroid, as the diameter upon which the sphere is described to the diameter upon which the spheroid is described. Now the contents of the segment of the circumscribed sphere is $(3t - 2h) h^2 \times .5236$. And, $(3t - 2h) h^2 \times .5236 : v :: t : k$; whence $v = .5236 (3t - 2h) \frac{k h^2}{t}$.

CIV. In this case we would have, $(3k - 2h) h^2 \times .5236 : v :: k : t$, whence $v = .5236 (3k - 2h) \frac{t k^2}{k}$.

CV. Let k be the fixed axis and t the revolving axis. Then with notation as in text, section 100, we have, for middle zone of sphere (XCVII) $(k^2 - \frac{1}{3} l^2) l \times .7854 = (3k^2 - l^2) l \times .2618$. But, $k^2 : t^2 :: (3k^2 - l^2) l \times .2618$

: v; and $k^2 : t^2 :: k^2 - l^2 : d^2$; whence $k^2 = \frac{t^2 l^2}{t^2 - d^2}$; then $\frac{t^2 l^2}{t^2 - d^2} : t^2 :: \left(\frac{3 t^2 l^2}{t^2 - d^2} - l^2\right) l \times .2618 : v$; whence $v = .2618\, l\, (2 t^2 + d^2)$.

CVI. This formula may be derived in manner similar to that adopted in CV.

CVII. Not supplied.

CVIII. See Davies' Dict. of Mathematics.

CIX. This is an average by means of a medium base.

CX. Davies' Dict. Mathematics.

CXI. Not supplied.

CXII. Not supplied.

ANSWERS TO PROBLEMS.

Exercise I.

(1), 88. (2), $22\frac{1}{2}$ sq. in. (3), 1092 sq. ch. (4), 65 sq. yds. (5), $97\frac{25}{48}$ sq. ft. (6), $10\frac{3}{8}$ in. (7), 21 ft. (8), 360 sq. perches.

Exercise II.

(1), 6. (2), $9\frac{1}{3}$ sq. yds. (3), 2.207 acres. (4), 58.7 sq. links. (5), 11.6 +. (6), 30, 40.

Exercise III.

(1), 6.93. (2), 89.34. (3), 13.26 acres. (4), 35.07 sq. ft. (5), 6.93. (6), 9.12.

Exercise IV.

(1), 5. (2), 4. (3), 3. (4), 241.8 ft. (5), 77. 8 + ft.

Exercise V.

(1), $s = 46.15+$. $s' = 32.05+$. $p' = 38.46$. (2), $p' = 22.77+$. (3), $h = 15.62+$ ft. $p' = 7.67+$ ft. $s = 9.21+$ ft., the distance from one side at which the prop must be placed.

General Exercise in the Mensuration of Triangles.

(1), 119.9 + ft. (2), 145.2 + yds. (3), $1\frac{1}{4}$ chains. (4), 173.2 +. (5), 209.76 + acres. (6), $5\frac{1}{4}$ acres. (7), 128 ft. (8), $42\frac{1}{4}$ ft. (9), $270\frac{1}{8}$ yds. (10), 19360 yds. (11), 4. (12), 80.83 + perches. (13), 316.6 ft. (14), 30 ft. (15), 157.-086 ft.

EXERCISE VI.

(1), 36. (2), 225 sq. chains. (3), $60\frac{1}{16}$ sq. ft. (4), 173.35+ acres.

EXERCISE VII.

(1), $12\frac{1}{4}$. (2), 72.

EXERCISE VIII.

(1), 198. (2), $42\frac{1}{8}$ sq. ft. (3), $15\frac{15}{16}$ acres. (4), $20\frac{1}{4}$ sq. ft. (5), 43.32. (6), 43.32 acres.

EXERCISE IX.

(1), $1\frac{1}{18}$ sq ft. (2), 6.51 acres.

EXERCISE X.

(1), $1\frac{5}{8}$ acres.

GENERAL EXERCISE IN THE MENSURATION OF QUADRILATERALS.

(1), 98.3 + perches. (2), 25. (3), 6.4 +. (4), $4\frac{39}{40}$ acres. (5), 35.92 + yds. (6), 96 sq. ft. (7), 11 A., 4 perches. (8), 2.91 acres.

EXERCISE XI.

(1), 48.28 + sq. in. (2), 49.73 sq. yds. (3), 1599.9 + yds. (4), 10. (5), 5.79 + acres.

EXERCISE XII.

(1), 6.87 acres.

EXERCISE XIII.

(1), 47.124. (2), 25.46 +. (3), 24856.34 miles. (4), 113.0976. (5), 1.4 sq. yds. (6), 24.97 acres. (7), 54.54 + sq. ft. (8), $39\frac{1}{4}$ yds. (9), 628.32. (10), 25 yds. (11), 19.635 sq. ft. (12), 3848.46.

EXERCISE XIV.

(1), 549.78. (2), 167.16 sq. ft. (3), 1649.34.

EXERCISE XV.

(1), 40⅜ ft. (2), 7½ ft. (3), 53½. (4), a = 24.99; k′ = 18.8 yds. (5), 40.8.

EXERCISE XVI.

(1), 236.9. (2), 385.13.

EXERCISE XVII.

(1), 143.43¾. (2), 24 sq. ft. (3), 1680 sq. yds. (4), 201.3.

EXERCISE XVIII.

(1), 26.59. (2), 22.57 yds. (3), 14.14. (4), 2.52 in. (5), 14.14 ft. (6), 44.43 ft. (7), 112.84. (8), 33.45.

EXERCISE XIX.

(1), 80.07 +. (2), 2748.9. (3), 1649.34. (4), 561.56 + sq. ft. (5), 19.1 +. (6), 10. (7), 7 or 28. (8), 25. (9), 40. (10), 210.

EXERCISE XX.

(1), 64. (2), 17.435. (3), 24.4+. (4), 240 acres. (5), 85⅝.

GENERAL PROBLEMS IN THE MENSURATION OF THE FIVE REGULAR BODIES.

(1), s=138.56; v=1396.28.
(2), s=2400; v=8000.
(3), s=110.85; v=60.34.
(4), s=2064.58; v=7663.10.
(5), s=419.91; v= 627.43.
(6), s=216.30 sq. yds.; v=272.71 cu. yds.

EXERCISE XXI.

(1), s=490 sq. in.; v=500 cu. in.
(2), 8460.2 gallons.

6 in. (35), 3184 sq. ft. (36), 750 sq. yds. (37), 112.83 per. (38), 27 a. 2 r. 8 per. 9 yds. (39), diameter, 914.76 chains; area, 135 a. 3 r. 6 per. 6 yds. (40), 14.9 minutes. (41), 157 ft. (42), 2.626 ft. (43), 62.83 ft. (44), $117.65. (45), 22-.136 in. (46), 27¼ yds. (47), 559.10 sq. in. (48), 26.3 in. (49), 23.24 in. (50), 27. (51), $54.30. (52), 8 in. (53), $1.18. (54), $53.36. (55), lower part, 2.528 in.; middle part, 3.605 in.; upper part, 13.867 in. (56), 1422.20 lbs. (57), 1st, .46 in.; 2d, .57 in.; 3d, .82 in., 4th, 3.149 in. (58). 55.56 miles. (59), 36,766 bbls., nearly. (60), 26½ cu. ft. (61), 148.34 ft. (62), 1527.05 sq. ft. (63), 523.95 sq. in. (64), 413.6124 ft. (65), 97.11 ft. (66) 276.63 yds. (67), 573.12 acres. (68), 39.27 sq. ft. (69), 4 in. (70), 30 ft. (71), 200 in. (72), 11.1 in. (73), 220 sq. in. (74), 27.71 in. (75), 6 in. (76), 37.92 sq. in. (77), v = 60.3 cu. in.; s = 110.85 sq. in. (78), 125 sq. in. (79), v = 67.81 cu. ft.; s = 56.88 sq. ft. (80), 22.34 sq. yds. (81), 83.14 cu. ft. (82), 62.83 sq. in. (83), .729 sq. in. (84), 9630 cu. in. (85), 14137.2 cu. in. (86), 50 ft. (87), 1507-97 cu. in. (88), 2513.28 cu. in. (89), 1058.33 pails. (90), 8.104 ft. (91), 916.48 yds. (92), 28.28 rods. (93), 35½ yds. (94), $72. (95), 77½ bu. (96), 231 cu. in.; 11 inches long, 7 inches wide, and 3 inches deep. (97), 18 ft., 3.349 in. (98), 64 rods. (99), 198¼ sq. rods. (100), 6 ft., 1 in. (101), 11½ yds. (102), 640 stones. (103), 12,000 yds. (104), $347.27. (105), $44.90. (106), $16.50. (107), $31.50. (108), 24 yds. (109), 21,780 cu. ft. (110), 21 ft. (111), 5½ feet. (112), 51.80 cu. ft. (113), 31$\frac{11}{16}$ cords. (114), 96 acres. (115), $59.25. (116), 2,940 feet. (117), 37 ft., 6 in. (118), 73 in. (119), 1½ yds. (120), 18 acres. (121) 3,456 gallons. (122), 6,415½ bricks. (123) $15.20. (124), $222.96. (125), 225 ft. (126), $12.94. (127), 27.7 ft. (128), 14$\frac{1}{2}$ cords. (129), $2,560. (130), 90$\frac{1}{8}$ yds. (131), 200.56 rods. (132), $48.21. (133), 91¼ cords. (134), 32 ft. (135), 22$\frac{1}{8}$ pieces; 72 yds. (136), 99.74 bbls.

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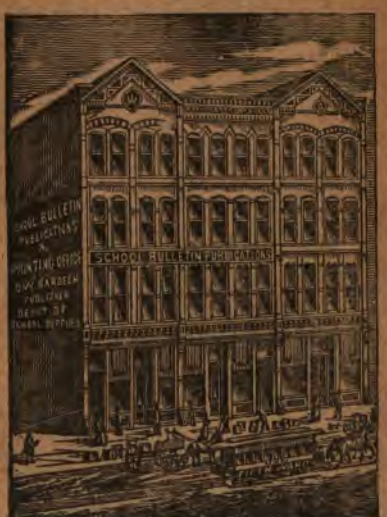

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